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Estimation of Tube Wall Compliance Using Pulse-Echo Acoustic Reflectometry

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Abstract—**This paper focuses on the estimation of tube wall compliance using reflection analysis of acoustic pulses. The wall compliance of a rubber latex tube was found theoretically using an acoustical transmission line model. Wall compliance was also obtained experimentally from acoustical and mechanical measurements. The acoustically estimated, mechanically estimated, and simulated wall compliances were C_{wExp}=6.55·10^{-7} cm^5/dyne, C_{wMech}=6.89·10^{-7} cm^5/dyne and C_{wSim} = 5.18·10^{-7} cm^5/dyne, respectively. The methods developed and the preliminary results obtained from this research could serve as the groundwork for the development of a device that determines the pathological condition of compliant biological conduits such as the airways.**

Keywords- **wall compliance, transmission line model, acoustic reflectometry.**

I. INTRODUCTION

Acoustic pulse reflectometry was originally developed for seismological studies involving the observation of stratifications in the Earth's crust. In the early seventies, Sondhi [1, 2] noted the potential use of acoustic pulse reflectometry as a method for measuring airway dimensions. In 1977, Jackson [3] published area-distance profiles of excised dog tracheas and lungs, measured using a pulse reflectometer. It was not until 1980 that Fredberg et.al [4] published the first area-distance profile measurements on human airways. Since then, acoustic reflectometry has been used as a noninvasive method to acoustically image human airways. Acoustic reflectometry has also been applied for the measurement of area-distance profiles of musical instruments [5, 6] and for the estimation of tube wall properties [7].

In this paper, we focus on the use of pulse-echo acoustic reflectometry, in conjunction with an acoustical transmission line model (TLM), to determine the wall compliance of a natural rubber latex pipe. Wall compliance was theoretically estimated using the TLM and compared to values measured experimentally via acoustic and mechanical methods. The work presented in this paper is novel since it employs transient acoustic pulses instead of continuous sinusoidal waves as presented in other publications [7].

A potential medical application of this technique is the estimation of airways' wall compliance, with the ultimate goal of detecting abnormal pathological changes in the human airways non-invasively. Respiratory diseases such as chronic obstructive pulmonary disease (COPD) and asthma cause inflammation of the airway wall, causing and increase in wall thickness and, therefore, a decrease in wall compliance.

II. THEORY

A. Acoustic Transmission Line Model

When sound waves travel in a tube with longitudinal dimensions that are comparable to the smallest wavelength of the sound wave in the tube, a distributed acoustical model must be used to represent sound propagation within the system. The approach used below is only valid if the sound waves traveling inside the system are planar (one dimensional propagation) and progressive (traveling in one direction as an incoming or outgoing wave). The acoustic transmission line model (TLM) has previously been used to model the acoustic response of the vocal [8] and respiratory tracts [9].

Figure 1 shows the electrical circuit representation of an incremental length of lossy cylindrical pipe with non-rigid walls. The acoustic resistance R_a represents viscous losses that occur at the tube-fluid interface. The conductance G_a models the heat conduction losses at the the tube walls. The inertance L_a is associated with the mass of the medium, while the compliance C_a is related to the ability of the medium to compress and expand. The wall impedance terms, C_w, R_w, and L_w are associated with the mechanical wall compliance, resistance, and mass, respectively. This wall impedance represents the wall mechanical opposition to wall motion. The mathematical expressions used to determine the transmission line circuit elements are shown in Table 1.

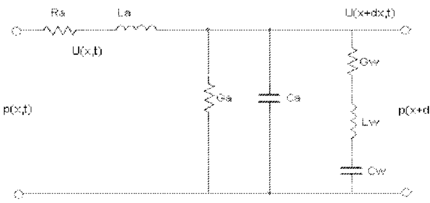

Fig. 1. Lumped model for acoustic-electrical models in one dimensional wave propagation on a short segment of pipe.

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TABLE I
YIELDING WALLS, LUMPED CIRCUIT PARAMETERS FOR A TUBE SEGMENT OF LENGTH L.

	Medium parameter	Wall Parameter
Viscous Friction	$R_a = \frac{2}{\pi r^3}\sqrt{\frac{w\rho_0\eta}{2}}$	$R_w = \frac{\eta_w h}{2\pi r^3 l}$
Compliance	$C_a = \frac{S}{\rho_0 c^2}$	$C_w = \frac{2\pi r^3 l}{Eh}$
Inertance	$L_a = \frac{\rho_0}{S}$	$L_w = \frac{\rho_w h}{2\pi r l}$
Heat conductance	$G_a = \frac{2\pi r(v-1)}{\rho_0 c^2}\sqrt{\frac{kw}{2c_p\rho_0}}$	

Sound pressure and volume velocity for plane wave propagation in a uniform tube satisfy the same wave equation as do voltage and current on a uniform transmission line. The general solution for the differential equation that defines the pressure at any given point of the transmission line is:

$$p(x,t) = Ae^{\gamma(j\omega)x}e^{j\omega t} + Be^{-\gamma(j\omega)x}e^{j\omega t} \qquad (1)$$

where A and B are constants determined by the system boundary conditions and $\gamma(\omega)$ is the propagation coefficient defined as:

$$\gamma(j\omega) = \sqrt{Z_a(j\omega)\left[Y_a(j\omega) + Y_w(j\omega)\right]} \qquad (2)$$

where Z_a, Y_a, and Y_w are the medium impedance, medium admittance, and wall admittance, respectively:

$$Z_a(j\omega) = R_a + j\omega L_a \qquad (3)$$

$$Y_a(j\omega) = G_a + j\omega C_a \qquad (4)$$

$$Y_w(j\omega) = \frac{1}{R_w + j\left[\omega L_w - \frac{1}{\omega C_w}\right]} \qquad (5)$$

The characteristic impedance $Z_c(j\omega)$ of the transmission line segment based on the electrical circuit representation shown in Figure 1 is

$$Z_c(j\omega) = \sqrt{\frac{Z_a(j\omega)}{Y_a(j\omega) + Y_w(j\omega)}} \qquad (6)$$

There are specific frequency values at which the magnitude curve of the characteristic acoustic impedance has maximum and minimum values [10]. The characteristic acoustic impedance reaches a minimum magnitude value at the wall resonance frequency f_{res}. Wall resonance occurs at the frequency at which the imaginary part of the wall impedance becomes zero. The wall resonance frequency f_{res} is given by

$$f_{res} = \frac{1}{2\pi}\sqrt{\frac{1}{L_w C_w}} = \frac{1}{2\pi r}\sqrt{\frac{E}{\rho_w}} \qquad (7)$$

There is an extended frequency limit f_2 where the characteristic impedance reaches a maximum [7]. An expression for f_2 is:

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{C_a + C_w}{L_w C_w C_a}} \qquad (8)$$

Combining equations (7) and (8), such that the wall inertance term L_w is eliminated, results in:

$$\frac{C_w}{C_a} = \left(\frac{f_2}{f_{res}}\right)^2 - 1 \qquad (9)$$

Equation (9) can be used to estimate the wall compliance C_w if one knows the medium compliance C_a and the values of f_{res} and f_2.

III. MATERIALS AND METHODS

A. *Computer Simulations*

Computer simulations, based on the acoustic TLM, were performed to find estimated values of C_w and for plotting the characteristic impedance of a natural rubber latex pipe. The circuit model elements for this particular configuration are shown in Table II. The simulated wall compliance value, denoted as C_{wSim}, was found using the expression for C_w shown in Table I. The wall compliance of the latex tube was estimated to be 5.18e-7 cm^5/dyne. Equation (6) was used to estimate the tube's characteristic impedance $Z_c(j\omega)$ as a function of frequency.

TABLE II
TRANSMISSION LINE MODEL PARAMETERS USED IN SIMULATION

	TLM Parameter	*Unit*
R_a	$6.1*10^{-3}$	*dyne*s/cm^6*
L_a	$3.6*10^{-3}$	*dyne*s^2/cm^6*
C_a	$2.3*10^{-7}$	*cm^5/dyne*
G_a	$1.91*10^{-7}$	*cm^5/dyne-s*
R_w	52.61	*dyne*s/cm^5*
L_w	$8.8*10^{-3}$	*dyne*s^2/cm^5*
C_w	$5.18*10^{-7}$	*cm^5/dyne*

B. *Acoustic Measurements*

Figure 2 depicts the experimental setup that was utilized for the acoustic measurements. The system consisted of a loudspeaker (Pyle PLM45) that was connected at one end of a 3.2 mm diameter copper pipe. A microphone (Knowles EM-3046) was connected such that its pressure sensing port was flush with the inner wall of the copper pipe. The electric signals used to drive the speaker were generated in a personal computer, amplified (Denon DRA-685) and sent to the speaker. Sound signals recorded at the microphone were amplified (AC-coupled, 40 dB gain) and then digitized with a data acquisition board (National Instruments PCI-MIO-16E-1). A virtual instrument (VI) was created using LabVIEW, and used for signal generation, acquisition, analysis and display.

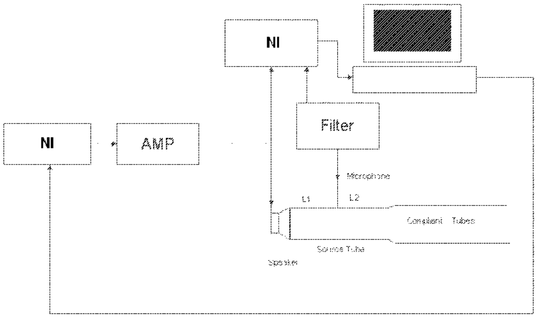

Fig. 2. This figure depicts the experimental setup utilized to perform the acoustic measurements.

A 3.2 mm diameter latex tube, was connected to the distal end of the acoustic reflectometer (copper pipe). The wall thickness of the rubber pipe was 190 µm. To perform an acoustic measurement, an acoustic pulse was generated by the loudspeaker. An *a priori* inverse filtering technique [10] was utilized to synthesize an acoustic Gaussian pulse whose frequency content ranged approximately from 20 Hz to 14 kHz. The resulting sound signal was acquired and analyzed to find the reflection coefficient $R(\omega)$ at the boundary between the copper and the rubber pipe. The reflection coefficient was found using

$$R(\omega) = \frac{P_r(\omega) \cdot e^{j\omega T_d}}{P_I(\omega)} \tag{10}$$

where $P_r(\omega)$ and $P_I(\omega)$ are the spectra of the reflected and incident sound signals, respectively, and T_d is the time delay associated to the round-trip travel time of the acoustic pulse from the microphone to the tube boundary (copper-rubber) and back. Since the rubber pipe was uniform and sufficiently long to be considered infinite, the characteristic acoustic impedance $Z_1(j\omega)$ of the rubber pipe was calculated with

$$Z_1(j\omega) = \frac{1+R(j\omega)}{1-R(j\omega)} \cdot Z_0 \tag{11}$$

where Z_0 is the characteristic impedance of the rigid copper pipe. Since the copper pipe is rigid, its acoustic characteristic impedance is independent of frequency, and can be found using

$$Z_0 = \frac{\rho_0 c}{S} \tag{12}$$

In equation (12), ρ_0 is the air density, c is the sound speed in air, and S is the cross-sectional area. The values of the frequencies f_{res} and f_2 can then be obtained from the characteristic impedance curve, and substituted in equation (9) to find the wall compliance, which in this case, was denoted C_{wExp}.

C. *Mechanical Measurements*

The static wall compliance of the latex rubber tube was measured mechanically. The procedure consisted of injecting a known volume of incompressible fluid (water) and measuring the resulting change in pressure. The volume was increased in small increments to obtain a pressure vs. volume curve. Care was taken not to exceed the elastic limit of the rubber latex material. The inverse of the slope of the pressure vs. volume curve was taken as the mechanically estimated wall compliance, denoted C_{wMech}.

IV. RESULTS

A. *Simulation*

Substitution of wall parameters into the TLM expression for C_w yielded a per unit length wall compliance $C_{wSim} = 5.18 \cdot 10^{-7}\ cm^5/dyne$.

B. *Mechanical Compliance Measurements*

The mechanically estimated wall compliance of the rubber latex tube was $C_{wMech} = 6.89 \cdot 10^{-7}\ cm^5/dyne$.

C. *Acoustic Measurements*

Figures 3 and 4 show the incident and reflected sound signals, respectively, as recorded by the microphone. The Fast Fourier Transform (FFT) representations of the incident, reflected and background noise signals are shown in Figure 5. It can be observed from this figure that, for frequency values below 1.5 kHz, the signal to noise ratio is low. Therefore, acoustic information contained at this lower frequency range was not suitable for analysis.

Figure 6 shows plots of simulated and experimental reflection coefficients, while Figure 7 depicts plots of simulated and experimental characteristic acoustic impedances. The minimum value of the Z_c occurred at a frequency f_{res}=2.85 kHz, while the maximum value occurred at a frequency of f_2=4.32 kHz. The experimental wall compliance was calculated using (9), yielding a wall compliance value of $C_{wExp} = 6.55 \cdot 10^{-7}$ cm^5/dyne.

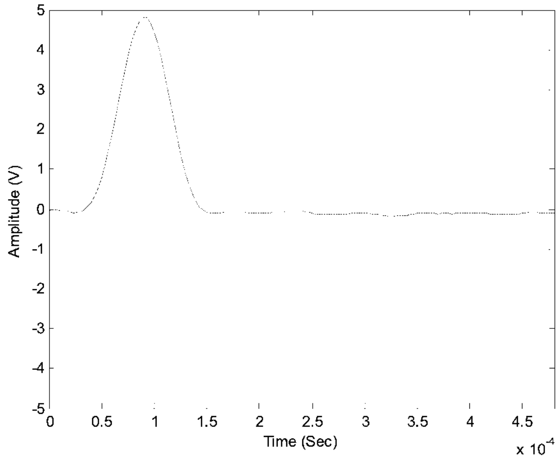

Fig. 3. Incident signal recorded at a sampling rate of 300 kS/s.

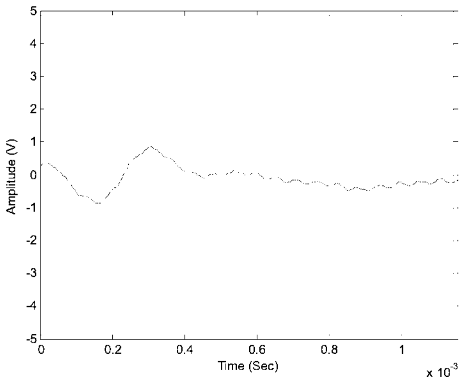

Fig. 4. Reflected signal recorded at a sampling rate of 300 kS/s.

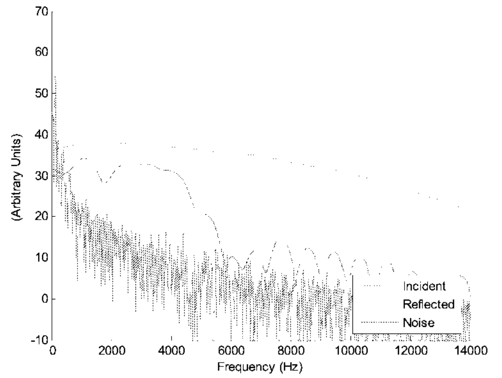

Fig. 5. Fast Fourier Transform of the incident, reflected and noise signal experimentally measured.

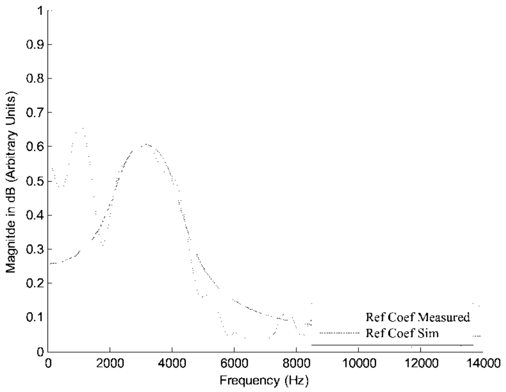

Fig. 6. Simulated and experimental reflection coefficient of the compliant tubes using time domain acoustic reflectometry.

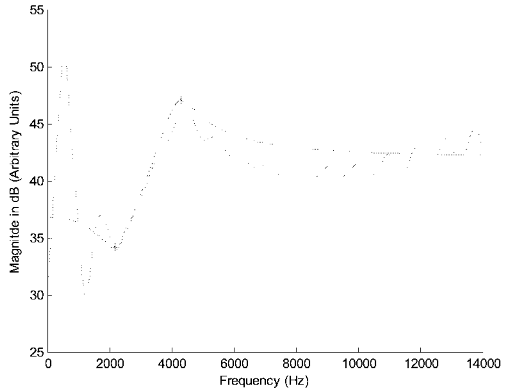

Fig. 7. Simulated (red) and measured (green) characteristic acoustic impedances.

V. DISCUSSION

Figure 7 depicts the characteristic impedance curves obtained from acoustic measurements and from the acoustical transmission line model. At frequencies higher than 1.5 kHz, where the signal to noise ratio was high, the experimental characteristic impedance curve closely resembled the one predicted by the acoustic transmission line model. In particular, the location of the valley and peak associated with the frequencies f_{res} and f_2, respectively, were comparable in both cases. Regarding the estimation of wall compliance, the acoustically estimated compliance value was in reasonable close agreement to those found mechanically and from simulations. The acoustically estimated, mechanically estimated, and simulated wall compliances were $C_{wExp}=6.55\cdot 10^{-7}$ cm^5/dyne, $C_{wMech}=6.89\cdot 10^{-7}$ cm^5/dyne and $C_{wSim} = 5.18\cdot 10^{-7}$ cm^5/dyne, respectively.

VI. CONCLUSIONS

Pulse-echo acoustic reflectometry was preliminarily used to estimate the wall compliance of a latex cylindrical conduit from measurements of its characteristic acoustic impedance. The methods described in this paper, which are based on an acoustical transmission line model, seem to be a promising technique to estimate the wall compliance of biological conduits non-invasively. More studies have to be performed, using different pipe materials and dimensions, to quantitatively describe the accuracy of this wall compliance measurement technique.

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