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Beamspace Magnetoencephalographic Signal Decomposition in Spherical Harmonics Domain

Tolga E. Özkurt, *Student Member, IEEE,* Mingui Sun, *Senior Member, IEEE,* and Robert J. Sclabassi, *Senior Member, IEEE*

***Abstract*— The recently proposed signal space separation (SSS) method can transform the multichannel magnetic measurements of brain (MEG) into parts that correspond to inner sources and undesired external interferences. In this paper, we extend this method by decomposing the signal into deep and superficial regions. This is realized by manipulating the SSS coefficients using a scheme that exploits beamspace methodology. It relies on estimating a linear transformation which maximizes the power of the source space of interest over the power of remaining part. We demonstrate that this method yields a simple and direct way to decompose the signal into deep and/or superficial parts.**

I. Introduction

Source estimation from magnetoencephalography (MEG) and electroencephalography (EEG) has been a significant problem. The well-known inverse solutions have been proposed for this purpose. However, before using the inverse solutions directly, pre-processing of data (such as artifact removing, dimension reduction, noise removal) may dramatically increase the performance in terms of both accuracy and computational efficiency. MEG signals have been transformed to different spaces for the purpose of both extracting useful information and dealing with computational burden. Techniques such as signal space projection [1,2], independent component analysis [3,4], adaptive filtering [5], beamspace techniques [6,7], SOFIA[8] and signal space separation [9,10] have been largely used in MEG data signal processing for one of both reasons. Among the pre-processing techniques, both the SSS and beamspace methods operate independently of dataset. This independence of transformation kernels prevents randomness coming from data and hence makes these methods statistically more reliable.

The beamspace processing relies on projecting data by maximizing the power with an orthogonal transformation matrix that spans the space of the lead field obtained by forward modeling computations. This pre-processing technique has recently been shown to increase the performance of main source localization algorithms [7]. Signal space separation (SSS), provided first by Taulu et al. [9] is a novel technique designed principally for removing interferences from MEG measurements. Since the sensor array is located in a current-free region, Laplace's equation becomes satisfied for magnetic scalar potential. Utilizing this fundamental law of physics, the SSS method decomposes the recorded magnetic field into two parts using vector spherical harmonic basis functions: one for the signals coming from the inside of the sensor array volume and the other coming from the outside of it. The method can effectively remove the external interferences without imposing unrealistic assumptions (unlike the conventional methods) by estimating the coefficients in least-linear square sense. This requirement arises from the inevitable non-uniqueness of the problem, i.e, one can find numerous solutions that can produce the same magnetic field on the MEG sensor array. Hence there have to be reasonable assumptions to achieve the decomposition either inside or outside of the head.

In this paper, we assume that the data are already separated to inner and outer parts via SSS and hence we focus on the decomposition of the inner part of the data. Decomposing the data into various regions can be vitally important for specific interesting components of the signal as in the case of a partial epileptic seizure originating from an unknown deep/superficial layer or left/right hemisphere of the brain. After obtaining the meaningful signal, one can proceed with the other analysis tools like source localization by standard inverse methods. The SSS method already gives a dimension reduced and interference-free signal that corresponds to inner part of the head. With an inspiration from beamspace methodology, we show how the SSS coefficients can provide a natural, useful and efficient method tool to focus on a region of interesting the head.

This paper is organized as follows. Section II describes the SSS and beamspace methods and illustrates the manipulations of the SSS coefficients necessary to constrain the data into deep/shallow parts. Section III demonstrates the results of numerical experiments realized with simulated data. Finally, the results and applicability of the proposed method are discussed in Section IV.

II. Algorithm Development

A. Beamspace with Signal Space Separation

For a noiseless environment, the mapping from source space to MEG data space is defined as

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The authors are with the Laboratory for Computational Neuroscience, Department of Neurological Surgery, University of Pittsburgh, Pittsburgh, PA, USA (e-mail: tolga@neuronet.pitt.edu).

$$B_k = \int_{\Omega} L_k(\mathbf{r'}) \cdot J(\mathbf{r'})\, d\Omega \quad (1)$$

where B_k is the measurement at the k^{th} coil, **r'** denotes the locations for whole source space Ω and J is the current source densities. A matrix that describes the second order relation between leadfields is called Gram matrix [6]

$$\mathbf{G} = \int_{\Omega} \mathbf{L}(\mathbf{r'})\mathbf{L}(\mathbf{r'})^T\, d\Omega \quad (2)$$

Where $\mathbf{L}=[L_1(\mathbf{r'})\ldots L_M(\mathbf{r'})]$ is the leadfield matrix for all sensors. The SSS method decomposes the magnetic signal at a sensor location **r** into a sum of two series expansions:

$$\mathbf{B}(\mathbf{r}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \alpha_{lm}\mathbf{x}_{\mathbf{lm}} + \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \beta_{lm}\mathbf{y}_{\mathbf{lm}} = \mathbf{B}_{in}(\mathbf{r}) + \mathbf{B}_{out}(\mathbf{r}) \quad (3)$$

using two different vector basis functions that are the gradients of spherical harmonic functions. For a radius R of the sensor array, these orthogonal basis functions separate the magnetic signal into two main parts that correspond to source locations **r'** where the magnitudes of the locations are |**r'**|<R and |**r'**|>R, respectively. If one rewrites the Eq. 3, with a matrix notation [9],

$$\mathbf{B} = \mathbf{S}^T\boldsymbol{\omega} \quad (4)$$

where basis functions matrix $\mathbf{S} = [\mathbf{S}_{in}\ \ \mathbf{S}_{out}]^T$ is comprised of inner and outer basis functions:

$$\mathbf{S}_{in} = [\mathbf{x}_{1,-1}, \mathbf{x}_{1,-1}, \mathbf{x}_{2,-2}, \ldots, \mathbf{x}_{L_{in},L_{in}}] \quad (5)$$

$$\mathbf{S}_{out} = [\mathbf{y}_{1,-1}, \mathbf{y}_{1,-1}, \mathbf{y}_{2,-2}, \ldots, \mathbf{y}_{L_{out},L_{out}}]$$

and the coefficient vector $\boldsymbol{\omega} = [\boldsymbol{\alpha}\ \boldsymbol{\beta}]^T$ contains the SSS coefficients for inner and outer parts:

$$\boldsymbol{\alpha} = [\alpha_{1,-1}, \alpha_{1,-1}, \alpha_{2,-2}, \ldots, \alpha_{L_{in},L_{in}}] \quad (6)$$

$$\boldsymbol{\beta} = [\beta_{1,-1}, \beta_{1,-1}, \beta_{2,-2}, \ldots, \beta_{L_{out},L_{out}}]$$

Note that the coefficients with $l=0$, $m=0$ are related to the so-called magnetic monopoles and are excluded from the SSS expansion. Our main goal is to extend this method by manipulating **α** to separate the signal inside of the head to the parts that correspond to |**r'**|< $\hat{r}$ and |**r'**|> $\hat{r}$ where $\hat{r}$ is any arbitrary radius which is less than R. Hence the signal will be decomposed into two parts: signal originating from the deeper sources and the signal from the shallow parts. This type of separation is illustrated in Fig. 1.

Beamspace method looks for a transformation matrix **T** that maximizes the power in a region of interest:

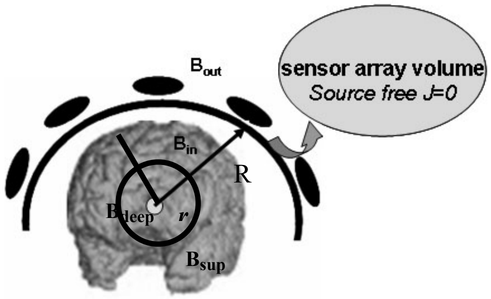

Fig 1. The SSS decomposes into B_{in} and B_{out}. The approach here decomposes into B_{deep} and B_{sup}

$$\max_{\mathbf{T}}\ tr(\mathbf{T}^T\mathbf{G}\mathbf{T}) \text{ with the constraint } \mathbf{T}^T\mathbf{T} = \mathbf{I} \quad (7)$$

Notice Eq. 7 implies maximizing the power in the magnetic signal without any assumption on the source current densities since they are completely unknown. Eq. 7 is equivalent to a typical eigenvalue problem and can be solved by obtaining **T** as the K eigenvectors of the Gram matrix **G** that correspond to the greatest K eigenvalues.

In order to deal with the same problem in the spherical harmonics domain, one should consider the leadfield like representations of the inner SSS coefficients given by [9]

$$\alpha_{lm} = \int_{\Omega} \lambda_{lm}(\mathbf{r'}) \cdot J_{in}(\mathbf{r'})\, d\Omega \quad (8)$$

where α_{lm}'s are the SSS inner part alpha coefficients and λ_{lm}'s are leadfield-like representations directly related to vector spherical harmonic function $\mathbf{X_{lm}}(\theta,\varphi)$

$$\boldsymbol{\lambda}_{\mathbf{lm}}(\mathbf{r'}) = \frac{i}{2l+1}\sqrt{\frac{l}{l+1}}\, r^l \mathbf{X}_{\mathbf{lm}}{}^*(\theta,\varphi) \quad (9)$$

$$\mathbf{X}_{\mathbf{lm}}(\theta,\varphi) = \frac{-1}{\sqrt{l(l+1)}}\left[\frac{mY_{lm}(\theta,\varphi)}{\sin\theta}\mathbf{e}_\theta + i\frac{\partial Y_{lm}(\theta,\varphi)}{\partial\theta}\mathbf{e}_\varphi\right] \quad (10)$$

Since MEG is sensible only to the tangential components of the source currents, $\mathbf{X_{lm}}(\theta,\varphi)$ does not have a radial component. Hence the dimension is naturally reduced from 3 to 2. Additionally, the vector spherical harmonics yield an orthogonal representation unlike the non-orthogonal representation by leadfield vectors of forward modeling computations. In this case, the Gram matrix for spherical harmonics domain can be defined as

$$G_{ij}^{S} = \int_{\Omega} \boldsymbol{\lambda}_{\mathbf{lm}}(\mathbf{r'}) \cdot \boldsymbol{\lambda}_{\mathbf{LM}}(\mathbf{r'})\, d\Omega$$

$$\mathbf{G}^S = \int_{\Omega} \boldsymbol{\Lambda}(\mathbf{r'})\boldsymbol{\Lambda}(\mathbf{r'})^H\, d\Omega \quad (11)$$

where $\boldsymbol{\Lambda}(\mathbf{r'}) = [\boldsymbol{\lambda}_{\mathbf{1\text{-}1}}(\mathbf{r'})\ \boldsymbol{\lambda}_{\mathbf{11}}(\mathbf{r'})\ \boldsymbol{\lambda}_{\mathbf{2\text{-}2}}(\mathbf{r'})\ldots\ \boldsymbol{\lambda}_{\mathbf{LL}}(\mathbf{r'})]^T$ whose dimension is (px2) and $p = (L_{in}+1)^2 - L_{in} - 1$. Notice that unlike in the classical Gram matrix, the sensor configuration is ot explicitly present anymore (since the ***α*** parameters already have this information with applying the SSS) and the dimension of the vectors is reduced from 3 to 2 (since the radial component of the sources is blind with a spherical form).

B. Methodology

Our methodology consists of finding a transformation in SSS domain that maximizes the power for the deep (or superficial) part while also minimizing the power for the superficial (or deep) part. This may be formulated as

$$\max_{\mathbf{T}} \frac{tr(\mathbf{T}^T\mathbf{G}_d\mathbf{T})/v_d}{tr(\mathbf{T}^T\mathbf{G}_s\mathbf{T})/v_s} \quad (12)$$

where $\mathbf{G}_d$ and $\mathbf{G}_s$ are Gram matrices; v_d and v_s are volumes for deep and superficial parts, respectively. The solution can be shown to be the largest eigenvectors of $\mathbf{G}_f = \frac{v_s}{v_d}\mathbf{G}_s^{-1/2}\mathbf{G}_d(\mathbf{G}_s^{-1/2})^T$.

For any l,m as the p^{th} SSS coefficient and L,M as the q^{th} SSS coefficient, the p^{th} row and q^{th} column of Gram matrix $\mathbf{G}_d$ for the deep part can be computed as

$$(\mathbf{G}_d)_{pq} = \iint_{\varphi\theta}\int_{r=0}^{\hat{r}}\left(\frac{i}{2l+1}\sqrt{\frac{l}{l+1}}r^l\mathbf{X}_{lm}*(\theta,\varphi)\right)$$
$$\times\left(\frac{-i}{2L+1}\sqrt{\frac{L}{L+1}}r^L\mathbf{X}_{LM}(\theta,\varphi)\right)r^2\sin\theta drd\theta d\varphi$$
$$=\delta_{lL}\delta_{mM}\frac{1}{(2l+1)^2}\frac{l}{l+1}\iint_{\varphi\theta}|\mathbf{X}_{lm}(\theta,\varphi)|^2\sin\theta drd\theta d\varphi$$
$$\times\int_{r=0}^{\hat{r}}r^{2l+2}dr \tag{13}$$
$$=\delta_{lL}\delta_{mM}\frac{1}{(2l+1)^2}\frac{l}{l+1}\frac{\hat{r}^{2l+3}}{2l+3}=:\delta_{pq}y(l)$$

Because of the orthogonality of the vector spherical harmonics, $\mathbf{G}_d$ will be diagonal:

$$\mathbf{G}_d = \begin{bmatrix} y(1) &&&&&&&&& \\ & y(1) &&&&&&&& \\ && y(2) &&&&&&& \\ &&& y(2) &&&&&& \\ &&&& y(2) &&&&& \\ &&&&& y(2) &&&& \\ &&&&&& \cdot &&& \\ &&&&&&& \cdot && \\ &&&&&&&& \cdot & \\ &&&&&&&&& y(L) \end{bmatrix} \tag{14}$$

We find $\mathbf{G}_s$ utilizing the similar computations above

$$(\mathbf{G}_s)_{pq} = \frac{1}{(2l+1)^2}\frac{l}{l+1}\frac{R^{2l+3}-\hat{r}^{2l+3}}{2l+3}\delta_{pq} \tag{15}$$

and finally the transformation matrix is diagonal :

$$(\mathbf{G}_f)_{pq} = \frac{v_s}{v_d}G_s^{-1/2}G_d(G_s^{-1/2})^T\delta_{pq} = \frac{v_s}{v_d}\frac{\hat{r}^{2l+3}}{R^{2l+3}-\hat{r}^{2l+3}}\delta_{pq} \tag{16}$$

The unity constraint $\mathbf{T}^T\mathbf{T}=\mathbf{I}$ in Eq. 7 is more meaningful for dimension reduction purposes (if one tried to satisfy the power constraint, **T** would be an identity matrix). Since the Gram matrix is diagonal and we want to preserve the power that belongs to the region of interest, we directly assign $\mathbf{G}_f$ to **T** instead of finding its eigenvectors. Hence decomposing the MEG signal to a part that corresponds to a deep spherical region in the brain is achieved by a simple manipulation of the SSS coefficients as

$$\tilde{\alpha}_{lm} = \alpha_{lm}\frac{v_s}{v_d}\frac{\hat{r}^{2l+3}}{R^{2l+3}-\hat{r}^{2l+3}} \tag{17}$$

and reconstructing the interesting signal with the obtained modified coefficients:

$$\hat{B}_{deep} = S_{in}\tilde{\boldsymbol{\alpha}} \tag{18}$$

III. Numerical Experiments

In order to verify the validity of the proposed method presented in Section II., we simulate some MEG data whose sensor characteristic is compatible with Elekta Neuromag® 306 channel system. This system measures the magnetic signal with 204 planar gradiometers and 102 magnetometers with a sampling frequency of 1 KHz. Two sources are assumed to exist in the brain: one as deep at (2,3,1) cm and the other considered as superficial at (6,5,6) cm. The radius of the head is $R=11$ cm. Deep source waveform is cosine-squared window (a peak between 200-300 ms) and the other is sinusoidal with high frequency. Some random Gaussian noise is added to simulate sensor noise.

$L_{in}=9$ and $L_{out}=1$ are chosen to estimate the SSS coefficients. Deep and superficial sources are estimated using the proposed algorithm by selecting the radius of the separating circle $\hat{r}=6$cm. This choice is reasonable since it is between the radius for the deep source is $r_d\approx 3.7$cm and for the superficial source $r_s\approx 9.8$cm.

The channel layouts for original signal and the separated deep part are supplied in Fig. 2. It is observed that the signal waveforms that belong to deep source and superficial sources are distinguished. Fig. 3 illustrates signals from a channel close to the deep source . It clearly distinguishes the superficial part which cannot be observed from the original simulated data. Additionally, a channel that is dominated with superficial source is exhibited in Fig. 4. The superficial part is also clearly separated as one can realize from the comparison of Fig. 4 (a) and 4 (b).

IV. Conclusion

The SSS algorithm obtains coefficients for interesting sources inside of the sensor array and the external interferences outside of it. We have showed that with a simple manipulation of the inner coefficients; the signal can be separated to parts that correspond to deep and superficial sources. This simplicity of the derived formulation comes from the natural appropriateness to spherical domain and orthogonality properties of the SSS basis functions that are directly related to the vector spherical harmonics. It is well-known that these functions are orthonormal eigenfunctions of the Laplacian operator on the spherical surface.

We utilized the beamspace methodology to modify the coefficients. It should be noted that it would not be easy to do these computations with classical lead field functions, i.e., since they are not orthogonal, one would have to first determine necessary coordinates by dividing the deep (or superficial) source volume into grids and compute the Gram matrix discretely unlike the proposed method. Moreover with our approach, while the beamspace transformation matrix is being obtained; one does not have to deal with the sensor configurations and dimension reduction, i.e, choosing the eigenvectors of the Gram matrix that correspond to the largest eigenvalues. All these procedures are already handled by the SSS method. Hence we use the beamformer only for decomposing in this study and for this particular separation, the diagonal Gram matrix can be directly assigned to the decomposing matrix.

Our study suggests a straightforward and efficient way to solve the separation problem inside of the head. For our future work, we plan to develop this algorithm to decompose the signal to different regions of interest such as left & right hemispheres and bottom & top parts of the head.

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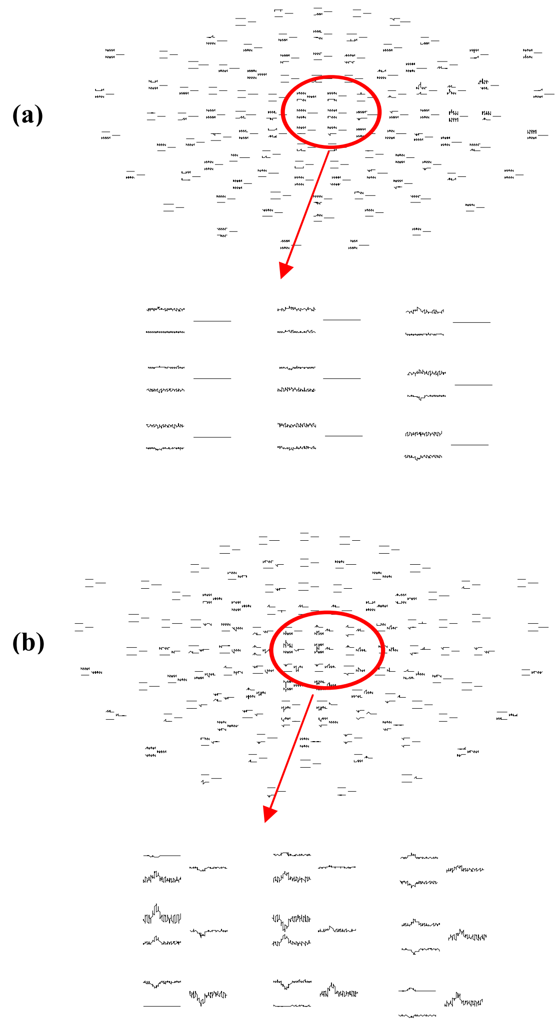

Fig. 2. Channel layouts for (a) total signal , (b) estimated deeper signal

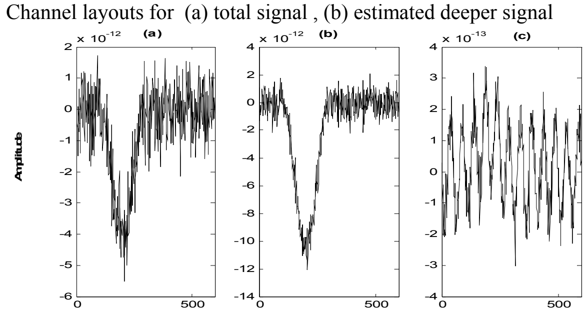

Fig. 3. (a) The signal that corresponds to inner sources (B_{in}), (b) Estimated deeper part, (c) Estimated superficial part for the 66th channel

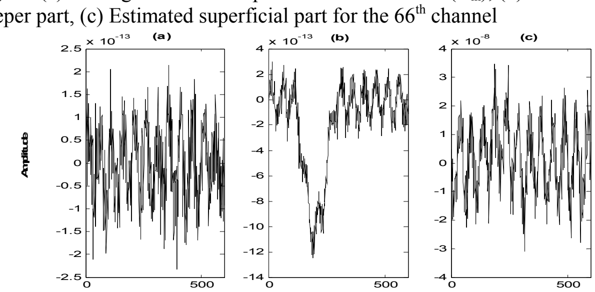

Fig. 4. (a) The signal that corresponds to inner sources (B_{in}), (b) Estimated deeper part, (c) Estimated superficial part for the 164th channel