Proceedings of the 28th IEEE
EMBS Annual International Conference
New York City, USA, Aug 30-Sept 3, 2006

Optimal Window Length in the Windowed Adaptive Chirplet Analysis of Visual Evoked Potentials

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Abstract—**Visual evoked potentials (VEPs) are electrical signals measured from the scalp in response to rapid and repetitive visual stimuli. The windowed adaptive chirplet transform (ACT) has been proposed recently to provide a unified and compact representation of VEPs from its transient portion to the steady-state portion. An important question concerns proper selection of window length. In this paper we show that the lower bound of the length is limited by the signal-to-noise ratio (SNR), while the upper bound is placed by the duration of the transient portion of VEPs. For our data, we have proposed an optimal length of 0.416 s (100 points). It is optimal in that under the condition of efficient estimators, the time-resolution of chirplet analysis is maintained as high as possible.**

I. INTRODUCTION

VISUAL voked potentials (VEPs) are surface electrical potentials measured from the scalp in response to a visual signal. It is believed that they are generated from the visual cortex and/or the peripheral neural pathways leading to the cortex and are time-locked to the visual stimulus [1]. One important type of VEP is steady-state VEPs (*ss*VEPs). An *ss*VEP is usually established if the repetition rate of visual stimuli is sufficiently high (usually, above 6 times per second [1]), and the responses begin to merge and the shape of the resulting VEP becomes periodic. A variety of clinical applications require the detection of *ss*VEPs. More recently, they have also found applications within interface design [2]. Therefore, reliable estimation and detection of *ss*VEP is desired.

With the assumption of steady-state, VEPs are usually modeled as a linear combination of a fundamental (usually, the stimulator) frequency and its higher harmonics. A detection task then is reduced to finding these periodic components embedded in the background spontaneous EEG [3]. However, this model is not always sufficient for the description of steady-state VEPs. Previous studies have shown that time is required for the formation of the steady-state response [4]. In general, two stages of the response can be observed – (1) a transient buildup portion preceding (2) the steady-state portion. Relying on just the conventional model, an experimenter will not be able to exploit the information contained in the transient VEP (*t*VEP) that appears immediately following the onset of the visual stimuli. Recently, a number of new applications require fast estimation and detection of VEPs (*e.g.* [2]. This has prompted us to find new ways to characterize the transient portion of the response.

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An emerging new technique based upon the adaptive chirplet transform (ACT) has been proposed to address these issues [5]. A chirplet is a time-varying frequency sweeping signal with a Gaussian envelope [6]. ACT decomposes VEPs into chirplet basis functions with four adjustable parameters (*i.e.* time-spread, chirp rate, time-center and frequency-center). The true essence of the adaptive ACT approach is to approximate a signal's energy curve in the time-frequency plane by using straight lines with arbitrary slopes (a first-order approximation). The results showed that it can provide a unified representation of the complete VEP responses (both transient and steady-state VEPs). Since the entire signal is under analysis at once, this approach is referred to as the non-windowed ACT. However, one major limitation of the non-windowed method comes from its difficulty in analyzing long-time persistent nonstationary signals that have continuous time-varying behavior over an indefinite time period. The reason of excessive cost of time is mainly due to the large data size, N, since chirplet estimation can be shown to have complexity $N^2 \log N$ [9]. Therefore, the time of computation will increase rapidly with the increase of data size. In some situations, however, minimum time cost is imperative. The non-windowed ACT approach usually cannot fulfill this purpose.

In order to reduce computational time, the approach of the windowed ACT was proposed [10]. The ACT was modified one step further by segmenting the signal as a preprocessing procedure. That is, the signal is partitioned into non-overlapping and equal length segments using rectangular truncation. In each windowed segment, a single chirplet is estimated and hence the entire signal is approximated by a sequence of chirplets. However, no convincing argument was provided to justify the selected window length. On the one hand, short window length makes the time-resolution of the analysis higher, but leads to large variance of estimation. On the other hand, longer window gives smaller variance, but introducing errors due to model mismatch.

In this paper we propose a method to obtain the optimal window length for VEP analysis. We will show that the upper and lower bounds of the length are decided by the characteristics of VEPs. An optimal length is selected in the sense that under the condition of efficient estimators, the time-resolution of the analysis is maintained as high as possible.

II. THE WINDOWED ACT

The chirplet transform has been under continual development since the early 1990s [6][7]. A Gaussian chirplet is a wave packet with a Gaussian envelope and four adjustable parameters

$$g_{t_c,\omega_c,c,\Delta_t} = \frac{1}{\sqrt{\sqrt{\pi}\Delta_t}} e^{-\frac{1}{2}\left(\frac{t-t_c}{\Delta_t}\right)^2} \times e^{j[c(t-t_c)+\omega_c](t-t_c)} \quad (1)$$

where $j=\sqrt{-1}$, t_c is the time-center in second, ω_c the frequency-center in rad, $\Delta_t > 0$ the scale that is the time-spread of a chirplet in second, and *c* the chirp rate in rad/s that characterizes the "quickness" of frequency changes. The chirplet transform is then defined as the inner product between a signal and the chirplet atom given in (1)

$$\begin{aligned} a_{t_c,\omega_c,c,\Delta_t} &= \langle f, g_I \rangle \\ &= \int_{-\infty}^{\infty} f(t)\, g^*_{t_c,\omega_c,c,\Delta_t}(t)\, dt \end{aligned} \quad (2)$$

The coefficients $a_{t_c,\omega_c,c,\Delta_t}$ reflect the similarity between the local structures of the signal and the chirplets. They represent the signal's energy content in a time-frequency region specified by the chirplets. The absolute value of a coefficient is the amplitude of the projection. The set of chirplet parameters is denoted by a continuous index set $I = (t_c, \omega_c c, \Delta_t)$.

Since the signal-to-noise ratio (SNR) of VEPs embedded in strong background noise and spontaneous EEG is rather low, the problem of VEP analysis becomes an estimation problem in principle. To apply the chirplet analysis, a signal model is implied as a weighted sum of chirplets

$$f(t) = \sum_{n=0}^{P-1} a_{I_n} g_{I_n}(t) + W(t) \quad (3)$$

where a_{I_n} is a complex weight of the selected chirplet $g_{I_n}(t)$, P is the number of chirplets, and $W(t)$ is a process of Gaussian white noise (GWN) with variance σ^2. Ideally, P chirplets should be chosen so as to minimize the difference $\left\| f - \sum_{n=0}^{P-1} a_{I_n} g_{I_n} \right\|$ globally. Unfortunately, this approach is generally considered unpractical as it is an *NP*-hard problem [8]. This means that no known polynomial time algorithm exists to solve this operation. To solve this problem, a suboptimal technique that based on the MP algorithm with chirplets [10] has been proposed [5]. It employs a coarse-refinement scheme to improve the SNR level of the estimation. Whenever a new chirplet g_{I_P} is estimated by the MP algorithm, all $P+1$ chirplets are progressively refined with the EM algorithm. This non-windowed approach is capable of characterizing the short *t*VEP and long *ss*VEP portions successfully.

Unfortunately, the computational cost of the non-windowed ACT is expensive. It has been shown that about 80 minutes was required to extract 10 chirplets from a 1200-point signal [9]. As mentioned earlier, a windowed approach has been proposed to lower the cost [11]. It is known that most time of computation is spent in selecting the optimal chirplet of the MP algorithm. The complex of this operation is $O(N^2 \log_2 N)$, where *N* is the signal size [10]. That is, if $C(N)$ is the number of operations, then $C(N) = k_1 N^2 \log_2 N$, where k_1 is a constant. Suppose the signal has been partitioned into *M* segments of length *L*. The number of operations of the windowed ACT will be $C_w(N) = k_2 L^2 \log_2 L$, where $k_2 = k_1 M$. The computational complex can then be reduced to $O(L^2 \log_2 L)$. Since L is usually much smaller than *N*, significant time of computing can be saved. But, how can we choose a proper window length?

III. Optimal Window Length

In this section we present the detailed procedure of finding the optimal window length according to the characteristics of VEP recordings. In general, the minimum variance of the estimates is decided by the SNR condition of the signal, assuming that the observational interval of the signal can be made infinitely long [12]. Given the finite intervals adopted in practice, therefore, the window length should be chosen to be longer than a minimal value. Otherwise, the variances of the estimates will significantly deviate from the minimum values and the estimators are no longer efficient (unbiased and minimum variance). Although a longer segment is usually expected to increase the quality of the estimates, a long window length in turn may fail to uncover a signal's time-dependent behavior. Therefore, an upper limit will be imposed by the desired "resolution" of analysis. That means the segment must be short enough for the chirplet features to adequately "sample" the time-dependent behavior of the signal spectrum, yet minimize the error in the estimate. We show next what the theoretical minimum variances in the chirplet estimation are and how the minimum window length should be chosen from analysis via simulation. The maximum allowable window length will then be given according to the duration of the transient portion of VEP.

Recall that in the windowed ACT only one chirplet is estimated from each of the segments with the MP algorithm. Consequently, we may use the single chirplet model to represent the windowed signal (*i.e.* $P=1$ in (3)). That is, the signal is modeled as:

$$f(t) = a g(t) + W(t) \quad (4)$$

$$a = A e^{-j\phi} \quad (5)$$

where the subscript *I* is omitted without confusion. In the MP algorithm, the four estimates of the chirplet parameters are obtained from

$$\begin{bmatrix} \hat{t} \\ \hat{\omega}_c \\ \hat{c} \\ \hat{\Delta}_t \end{bmatrix} = \arg \max_{t_c,\omega_c,c,\Delta_t} \left| \langle f, g_I \rangle \right|^2 \quad (6)$$

where g_I is one of the predefined chirplets contained in a "dictionary" [8]. Subsequently, the estimated chirplet can be found by substituting (6) into (1). The other estimates are found by

$$\begin{aligned} \hat{A} &= |\hat{z}| \\ \hat{\phi} &= -\measuredangle \hat{z} \\ \hat{\sigma}^2 &= \frac{\|f\|^2 - |\hat{z}|^2}{2L} \end{aligned} \quad (7)$$

where *L* is the size of $f(t)$ and $\hat{z} = \langle f, g \rangle$. It can be shown that the estimators (6) and (7) are asymptotically unbiased, and that their minimum variance can be found by calculating the Cramér-Rao lower bounds (CRLBs) [12]. Fortunately, the bounds can be found in closed form. Only the bounds of the

TABLE I
CRLBS OF THE CHIRPLET ESTIMATES

$\hat{\theta}$	$\hat{t}_c$	$\hat{\omega}_c$	$\hat{c}$	$\hat{\Delta}_t$
CRLB	$\frac{\Delta_t^2}{\xi}$	$\frac{1+4c^2\Delta_t^4}{\xi\Delta_t^2}$	$\frac{4}{\xi\Delta_t^4}$	$\frac{\Delta_t^2}{2\xi}$

$\xi = A^2/2\sigma^2$

chirplet estimates are summarized in Table I. Interested readers are referred to [9] (cf. [13]) for the complete results and derivations.

The performance of the actual estimators was evaluated by computer simulation. The parameters of the model for simulation were $A=1$, $\phi=0$, $t_c=L/2$, $\omega_c=\pi/2$, $c=\pi/L$ and $\Delta_t=L/50$, where the window length L varied from 24 points to 1000 points. The variances of GWN σ^2 are chosen in such a way that the SNR of the simulated signal ranges from -20 dB to 20 dB. The SNR is defined as

$$SNR = 10\log_{10}\frac{A^2}{2(L-1)\sigma^2}. \quad (8)$$

The results are shown in Fig. 1 where the mean square errors (MSE) of the parameter estimates are compared with the CRLBs. They display in general two characteristics: one is that MSE approaches CRLB with the increase of SNR and follows it closely at high SNR levels; the other is that longer window length makes MSE converge to CRLB at lower SNR. We have estimated the SNR level of our data (see Section IV) that the average SNR is 0.55±0.98 dB, slightly above 0 dB [9]. We can see from Fig. 1 that at SNR=0 dB the MSE of the estimates will follow the CRLB closely when the window length is not less than 100 points. The results indicate that given VEP SNR at 0 dB, the window length should not be lower than 100 points in order to keep the estimators efficient. This helps determine the lower bound of the window length.

On the other hand, a segment must be short enough to adequately sample the energy density curves of a signal in the time-frequency plane. As for a VEP response, since most of the variations occur in the transients, a choice of the upper bound will be mainly influenced by the time duration of the transient portion of the signal, *i.e.* *t*VEP. In order to reveal sufficiently the time-dependent behavior of the signal in this portion, usually a window length that is shorter than half of the duration is employed. We propose the following method to estimate the duration of *t*VEP (Fig. 2): (1) Reconstruct the *t*VEP signal from the chirplets extracted by the non-windowed ACT [9]; (2) Find the envelope of the tVEP, which is defined as the square-root values of the signal and its Hilbert transform, *i.e.* $\sqrt{s^2(t)+H[s(t)]^2}$ where $s(t)$ is the real *t*VEP signal and $H[s(t)]$ is its Hilbert transform; (3) Subsequently, find the maximum of the envelope $A>0$ and the threshold is arbitrarily set at A/100; and (4) Finally, define the duration time as the time interval between the instants at which the first point and the last point on the envelope are above the threshold. According to this procedure, we found that the average duration of our data is 300±38 points. Thus, a window length should be smaller than 150 points.

Considering the lower limit imposed by the SNR condition, we therefore choose 100 points (416.7 ms at sampling rate of

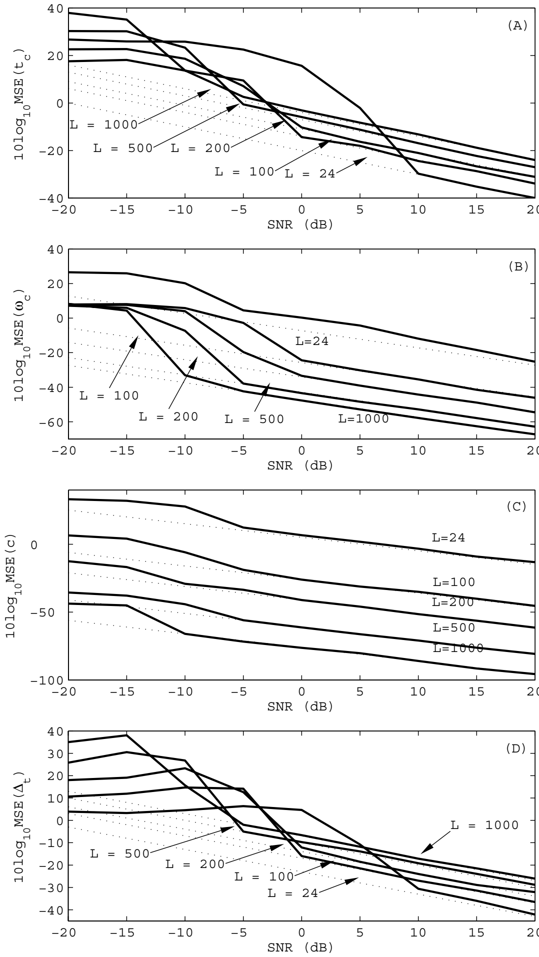

Fig. 1. Simulation results of estimating a single chirplet in window lengths and SNR levels. In each figure the dotted lines and solid lines are respectively the CRLBs and MSE of the estimates of corresponding estimators: (A) time-center ($\hat{t}_c$), (B) frequency-center ($\hat{\omega}_c$), (C) chirp rate ($\hat{c}$) and (D) time-spread ($\hat{\Delta}_t$).

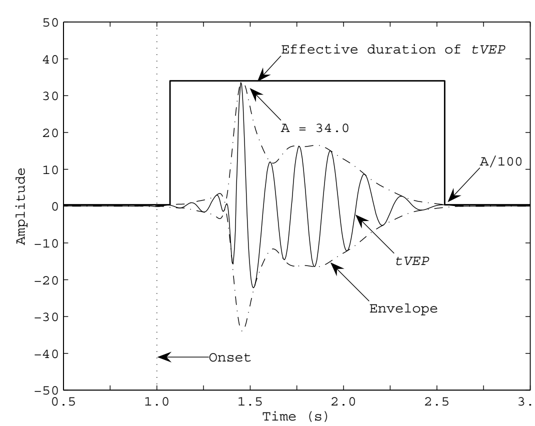

Fig. 2. Time duration of *t*VEP. The envelope of the reconstructed *t*VEP (of signal D_1) is shown in the dot-dash lines. Its maximum is denoted by A (34.0 in this example). The time duration of the *t*VEP is illustrated by a rectangular window. Its starting time and ending time are the instants when the first and last values on the envelope are greater than A/100 (or 0.340).

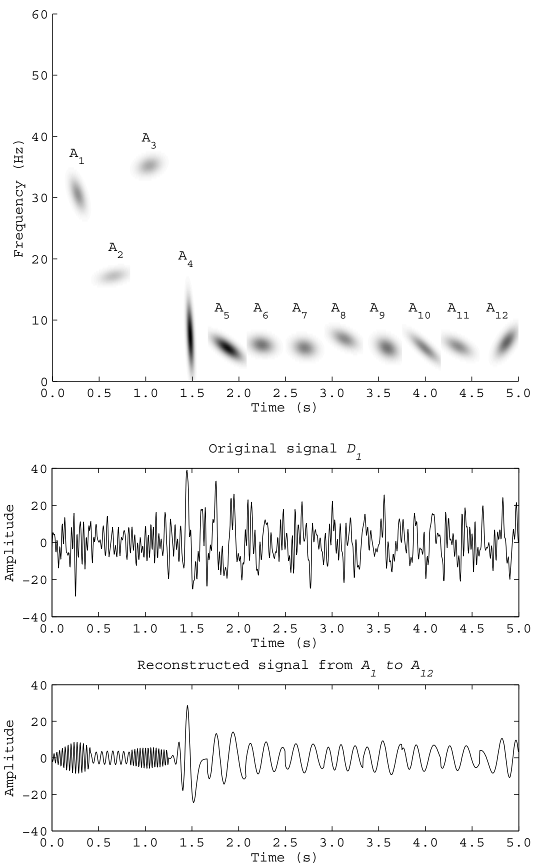

Fig. 3. Time-frequency structures of signals D_1. (A) 12 estimated chirplets with the windowed ACT (labeled as $A_1 - A_{12}$). (B) The original signal D_1 and reconstructed signal. Note that the black triangles indicate the onset of stimulation.

240 Hz) as the window length for partition and thus keep the time-resolution as high as possible.

IV. Results and Discussion

The VEPs are motion evoked potentials recorded at the Oz position of 10-20 international system above the primary visual cortex. The data were sampled at 240 Hz for 5 seconds before lowpass-filtered at 40 Hz (-3 dB). The onset of visual stimulation was at one second. In total, we obtained 5 signals denoted as $D_1 - D_5$ [5]. Since the window length was 100 points, 12 chirplets were estimated from each signal. As an example, the results of applying the windowed ACT to signal D_1 are shown in Fig. 3, together with the original and the reconstructed signals. We can see that the frequency-centers of the first three chirplets $A_1 - A_3$ are significantly higher than those of the remaining chirplets. The signal represented by these three chirplets is usually spontaneous EEG activity. This is followed by two chirplets, A_4 and A_5, with relatively higher amplitude. Specially, we note that A_4 acquires small time-spread (0.04 s) and large chirp rate (-25.77 Hz/s), indicating a decrease of instantaneous frequency. Chirplets A_4 and A_5 appear to represent the transient VEP portion. The rest of the signal is represented by the chirplets $A_6 - A_7$ with relatively stable amplitudes, frequency-centers, chirp rates and time-spreads, indicating the steady-state portion of a VEP response. Similar results were obtained from other signals $D_2 - D_5$.

One question concerning the variation of the estimates caused by the time shift of window positions deserves further discussion. In practice, the EEG signals usually are monitored continuously. Because the arrival time of a VEP response is usually unpredictable, the relative window positions are not necessarily the same as those shown in Fig. 3. To verify the effectiveness of the proposed window length, we shifted the window positions by half window length, *i.e.* 0.21 s, compared to Fig. 3. The results showed that the transient portions *t*VEP could still be clearly represented by two chirplets, which were followed by a series of chirplets representing the *ss*VEP [9]. From above discussions, we conclude that the window length of 0.42 s (100 points) is effective in VEP analysis for our data by the windowed ACT method.

V. Summary

In this paper we proposed a method to decide the optimal window length for chirplet analysis of VEP signals. We pointed out that two characteristics of a VEP signal determine the bounds of the length. The lower bound is placed by the SNR level of the signal, while the upper bound is limited by the duration of the transient portion of VEP. These results can be useful for further improvement of VEP analysis with chirplet representation.

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