

# Estimation of Oxygen Consumption for Moderate Exercises by Using a Hammerstein Model

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**Abstract**—This paper aims to establish block-structured nonlinear model (Hammerstein model) to predict oxygen uptake during moderate treadmill exercises. In order to model the steady state relationship between oxygen uptake (oxygen consumption) and walking speed, six healthy male subjects walked on a motor driven treadmill at six different speed (2,3,4,5,6, and 7 km/h). The averaged oxygen uptake of exercisers at steady state was measured by a mixing chamber based gas analyzer(AEI Moxus Metabolic Cart). Based on these reliable experiment data, a nonlinear static function was obtained by using Support Vector Regression. In order to capture the dynamics of oxygen uptake, a suitable Pseudo Random Binary Signal (PRBS) input was designed and implemented on a computer controlled treadmill. Breath by breath analysis of all exercisers' dynamic responses (PRBS responses) to treadmill walking was performed. A useful ARX model is identified to justify the measured oxygen uptake dynamics within the aerobic range. Finally, a Hammerstein is achieved, which is useful for the control system design of oxygen uptake regulation during treadmill exercises.

**Index Terms**—Oxygen uptake; Support Vector Regression; Hammerstein model; identification; treadmill exercise; PRBS.

## I. INTRODUCTION

OXYGEN uptake is an important physiological parameter for the determination of functional health status and clinical assessments in normal and pathological conditions. The main goal of this paper is to establish practical models to dynamically estimate oxygen uptake for walking exercisers. These models are also potentially applicable for the regulation of oxygen uptake during treadmill exercise.

Papers about oxygen uptake modeling for moderate exercise can be divided into two categories: oxygen uptake estimation at steady state condition (oxygen consumption  $\text{VO}_2$ ) and dynamic response characterization during onset and offset exercises. For the prediction of steady state oxygen uptake, Franklin et al [1] proposed a linear static model to approximately estimate oxygen consumption in a given walking speed ranges. Dill et al [2] provided simple static nonlinear (polynomial) models.

The oxygen uptake kinetic description at the onset and offset of leg work was first reported by Hill and Lupton [3] in 1923. After that, many papers attempt to capture both the exact time course of these kinetic adjustments and

the physiological events governing the rate of adjustment. Most of them [4] [5] approximate the process by using first order linear models, which were often obtained by using step change responses. Hoffmann et al [6] and Efeld et al [7] applied PRBS and sine signals to bicycle exercises. By using spectrum analysis, they proved the dynamic linearity of oxygen uptake with work load at low frequency.

In this paper, the steady state and dynamics of oxygen uptake during treadmill walking will be capture by using only one Hammerstein model. A novel model identification method for Hammerstein model will also be presented. The Hammerstein model can be described as a static nonlinear block followed by a dynamic linear system. Hammerstein models may account for nonlinear effects encountered in not only industrial processes [8], but also physiological processes. For example, lung tissue strip mechanics [9] can be effectively modeled by Hammerstein models.

The steady state relationship between treadmill speed and exercise workload has been found in our laboratory to be nonlinear. The dynamic linearity of oxygen uptake with workload within aerobic range has been proved in [6]. Therefore, it is reasonable to describe the relationship between treadmill speed and oxygen uptake by a nonlinear static element cascaded by a linear dynamic element, a Hammerstein model.

The modeling of Hammerstein model is a hot research topic. Recently, Goethals et al [10] presented a novel overparametrization (two-stage procedures) identification approach for Hammerstein systems. The most distinguish part of that approach is the utility of a powerful machine learning method, Least Square Support Vector Machine (LS-SVM). This novel machine learning method sets that paper apart from existing papers.

Support Vector Machine based regression (Support Vector Regression (SVR)) is a new technique, which has been successfully applied to nonlinear function estimation. This paper applies SVM approaches combining with stochastic method [11] to identify physiological processes. There are at least two aspects which are different with [10]:

The stochastic method [11] is employed in preference to the over parameterization method [12]. As discussed in [11], the error of the identification of Hammerstein model is not only from linear and nonlinear parts themselves but also from the coupling between them. The pseudo-random binary sequences (PRBS) are applied to decouple the identification of the two parts as suggested in [11]. Another main difference of this approach with [10] is the usage of  $\epsilon$ -insensitivity SVM [13] instead of LS-SVM (Least Square SVM) [14]. Both LS-

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SVM and  $\epsilon$ -insensitivity SVM have the merits of SVM approaches. However, the loss function used by  $\epsilon$ -insensitivity SVM, only penalizes errors greater than a threshold  $\epsilon$ . This leads to a sparse representation of the decision rule giving significant algorithmic and representation advantages [13]. On the other hand, the ridge regression ( $\epsilon = 0$ ) used by LS-SVM typically causes the loss of sparseness representation.

It should be emphasized that the proposed Hammerstein model identification approach is especially suitable for the identification of oxygen uptake model. For steady state oxygen uptake measurement, the most suitable way is to employ a mixing chamber to interrupt gas flow and thus prevent streaming of gases and uneven gas concentrations. However, gas samples collected from a mixing chamber are averaged gas fractions over time. The sensitivity to changes in oxygen uptake is therefore reduced [15]. In order to capture dynamical characteristics, breath by breath analysis should be applied for dynamical experiments. However, this may lead to the degrading of measurement accuracy in steady state test. Due to the totally decoupling of static part identification (based on steady state experiment data) and dynamic part identification (based on dynamic experiment data), it is permitted to apply different measurement approaches for steady state tests and dynamic experiments respectively. Therefore, a better estimation result is attainable.

The paper is organized as follows. The details of SVM based Hammerstein model identification approach is given in Section 2. Section 3 presents the application of the proposed approach for the modeling of oxygen uptake during treadmill exercises.

## II. PROPOSED SVM BASED HAMMERSTEIN MODEL IDENTIFICATION APPROACH

In [11], Bai showed that the identification of linear part of a Hammerstein model can be decoupled from nonlinear part with the help of the PRBS input. The reason is any static nonlinearity can be exactly characterized by a linear function under PRBS input which has the binary nature. In this study, the PRBS input is also employed. Thus, the identification of Hammerstein model can be obtained by the identification of static nonlinearity and linear dynamic separately.

As suggested in [11], the steady state gain of the linear dynamic model is constrained to be unity. The steady state characteristic of the Hammerstein system is considered by the static nonlinearity. It should be mentioned that for the identification of the static nonlinearity part, steady state experiments should be made because the PRBS inputs often do not excite the nonlinearity sufficiently in our interested range. Now, we introduce the so called  $\epsilon$ -insensitivity SVR based static nonlinearity modeling first [16].

Let  $\{u_i, y_i\}_{i=1}^N \subseteq \mathcal{R}^d \times \mathcal{R}$  be the inputs and outputs of the data of a Hammerstein system measured in steady state. The goal of the support vector regression is to find a function  $f(u)$  which has the following form

$$f(u) = w \cdot \phi(u) + b, \quad (1)$$

where  $\phi(u)$  represents the high-dimensional feature spaces which are nonlinearly transformed from  $u$ . The coefficients  $w$  and  $b$  are estimated by minimizing the regularized risk function:

$$\frac{1}{2} \|w\|^2 + C \frac{1}{N} \sum_{i=1}^N L_\epsilon(y_i, f(u_i)). \quad (2)$$

The first term is called the regularized term. The second term is the empirical error measured by  $\epsilon$ -insensitivity loss function which is defined as:

$$L_\epsilon(y_i, f(u_i)) = \begin{cases} |y_i - f(u_i)| - \epsilon, & |y_i - f(u_i)| > \epsilon \\ 0, & |y_i - f(u_i)| \leq \epsilon \end{cases} \quad (3)$$

This defines a  $\epsilon$  tube. The radius  $\epsilon$  of the tube and the regularization constant  $C$  are both determined by user.

By solving this constrained optimization problem, we have

$$f(u) = \sum_{i=1}^N \beta_i \phi(u_i) \cdot \phi(u) + b. \quad (4)$$

As mentioned before, by the use of kernels, all necessary computations can be performed directly in input space, without having to compute the map  $\phi(u)$  explicitly. After introducing kernel function  $k(u_i, u_j)$ , the above equation can be rewritten as follows.

$$f(u) = \sum_{i=1}^N \beta_i k(u_i, u) + b. \quad (5)$$

Where the coefficients  $\beta_i$  corresponding to each  $(u_i, y_i)$ , and only the so-called support vectors can have nonzero coefficients.

For linear support regression, the kernel function is thus the inner product in the input space:

$$f(u) = \sum_{i=1}^N \beta_i \langle u_i, u \rangle + b. \quad (6)$$

For nonlinear SVR, there are a number of kernel functions which have been found to provide good generalization capabilities, such as polynomials, Radial basis function (RBF), sigmod. Here we give the polynomials and RBF kernel functions as follows:

RBF kernel:  $k(u, u') = \exp(-\frac{\|u-u'\|^2}{2\sigma^2})$ ,  
 Polynomial kernel:  $k(u, u') = ((u \cdot u') + b)^d$ .

Detailed discussion about SVR, such as the selection of radius  $\epsilon$  of the tube, kernel function, and the regularization constant  $C$ , can be found in [16].

As PRBS input is employed to the identification of Hammerstein system, as shown in equation (2.3) of [11], the identification of a Hammerstein model can be simplified as a linear identification problem. Any linear identification approach can be applied. Here, the parametric approach as suggested in [11] is adopted.

## III. MODELING OF OXYGEN UPTAKE BY USING A HAMMERSTEIN MODEL

In this section, a Hammerstein model will be set up to estimate oxygen uptake from walking speed by using the introduced approach.

### A. Experiment equipments

The computer controlled treadmill and its related data collection and processing system are shown in Fig. 1.

The treadmill used in the system is the Powerjog “G” Series fully motorized medical grade treadmill manufactured by Sport Engineering Limited, England. Control of the treadmill can be achieved through an RS232 serial port. The treadmill can receive commands from the computer controller via this link, and obeys such commands without supervision. In order to implement PRBS type signal on treadmill, a computer based control system is implemented which can control the speed of treadmill with satisfactory dynamics. During experiments, all signals are synchronized with the PRBS signal. The synchronization of signals was monitored by triaxial accelerometers.

The measurement of oxygen uptake (either averaged with mixing chamber or breath by breath recorded) is implemented by using AEI Moxus Metabolic Cart.

### B. Nonlinear component modeling by using Support Vector Regression

In order to identify the nonlinear relationship, steady state experiments are performed. Six young healthy male subjects volunteered to participate in the study. Their physical characteristics are presented in Table I.

	Mean	SD	Range
Age (yr)	31.61	5.78	23-37
Height (cm)	176.41	5.48	169-184
Body mass (kg)	74.31	9.35	60-85

TABLE I  
SUBJECT CHARACTERISTICS (N=6)

All experiments were conducted in the afternoon, and the subjects were permitted to have a light meal one hour before measurements were recorded. Initially, the subjects were asked to walk for about 10 minutes on the treadmill to familiarize them with the experiment. The subjects were then requested to walk at six levels of different speeds (2, 3, 4, 5, 6 and 7 km/h). Each level took a total period of 5 minutes, and was followed by a 10-minute resting period. The oxygen uptake was recorded and averaged in every two minutes by using a mixing chamber based gas analyzer and ventilation measurement system (AEI Moxus Metabolic Cart). Finally, in order to identify linear dynamic part of the Hammerstein system, subjects were also requested to walk on the treadmill under a PRBS input ( $a_+ = 6km/h$  and

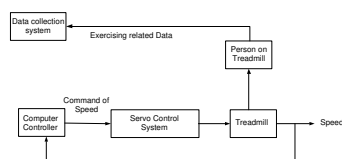


Fig. 1. Block diagram for experimental settings

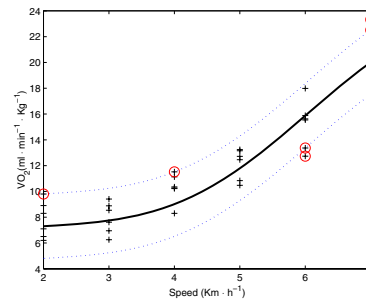


Fig. 2. The steady state relationship captured by using SVR

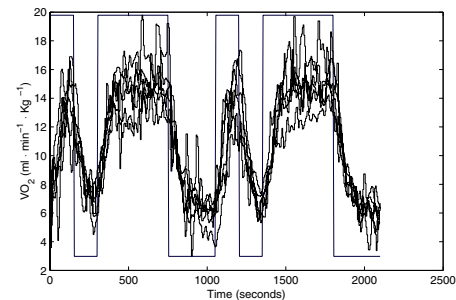


Fig. 3. The oxygen uptake of all six subjects under a PRBS input

$a_- = 4km/h$ ). Throughout the experiments, the breath by breath tidal volume and the concentration of oxygen were recorded to calculate breath by breath oxygen uptake. The outputs of triaxial accelerometers (mounted on the lower back of the subjects) were also recorded.

In this paper, both traditional linear regression and the  $\epsilon$ -insensitivity SVR regression method are applied to modeling the nonlinear part. The regression error (Root Mean Square error) of SVR (1.66) is much smaller than linear regression error (2.17). The SVR regression results are summarized in Table II and Fig. 2.

Kernel	Parameter	Constant C
RBF	$\sigma=3$	50
$\epsilon$ -insensitivity	Support vectors number	RMS error
2.5	6	1.66
$(ml \cdot min^{-1} \cdot Kg^{-1})$	(17.1 %)	

TABLE II  
DETAILS ABOUT THE ESTIMATION OF THE NONLINEARITY BY SVR

### C. Modeling for linear dynamic part

The oxygen uptake of all six subjects under a PRBS input is shown in Fig. 3.

The averaged oxygen uptake of six experiments is shown in Fig. 4. Papers [3] [17] often select first order exponential, with no time delays to describe the dynamics of oxygen uptake. In this study, we also adopt first order exponential to describe the dynamics. However, the time delays will be explored in this study because time delay is a crucial

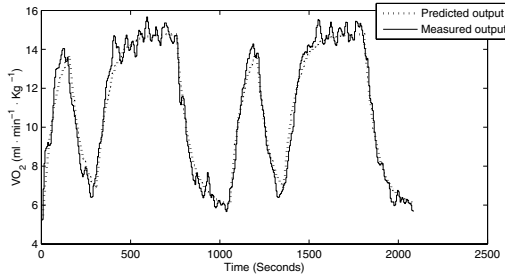


Fig. 4. Model estimation results

parameter for control system design. Neglecting of time delays often introduces instability to the controlled closed loop. In this study, the triaxial accelerometers are employed to synchronize body movements with respiratory related signals, which ensures reliable estimation of time delays.

Based on the averaged data of PRBS input experiments, the time delay is determined by using Matlab Identification Toolbox. Two popular model selections criteria, Minimum Description Length (MDL) and Akaike Information Criterion (AIC), select the linear model as follows:

$$y(k) = 0.9308y(k-1) + 0.0692u(k-1) + e(k), \quad (7)$$

with sampling period  $T_s = 5$  seconds. The noise variance of model (7) is  $3.5 \times 10^{-4}$ . By transferring model (7) to its corresponding continuous form (8)

$$Y(s) = \frac{1}{69.735s + 1}U(s), \quad (8)$$

we can see that the dynamic part is a first order stable system (with time constant  $T = 69.735$  seconds and unit steady state gain) without time delay. This supports the belief [3] [17] that the exponential rise in oxygen uptake directly reflects the rate of rise and drop in leg muscle oxygen consumption ( $\dot{Q}O_2$ ) at the onset and offset of exercises. The model fitting result is shown in Figure 4. Time constant and noise variance of model for each subject are summarized in Table III.

Subjects	1	2	3	4	5	6
Time constant (Seconds)	45.4	61.8	69.3	63.3	65.8	64.1
Noise variance of models	0.0013	0.0018	0.002	0.0017	0.0015	0.002

TABLE III  
TIME CONSTANT AND NOISE VARIANCE OF MODELS

Although the fitness of subjects is only determined based on our experiences rather than rigors assessment, a conjecture is proposed based on the calculated time constant (see Table III): fit subjects (subject 1) may have fast response (smaller time constants) with increasing or decreasing of workload.

#### IV. CONCLUSION

This paper provides a Hammerstein model identification method to estimate oxygen uptake for treadmill walking

exercises. The identification of dynamic linear component is decoupled by using a PRBS signal. In order to obtain good modeling results, breath by breath analysis is performed for dynamical experiments with PRBS input. It is found that the time constants of identified ARX models for increasing and decreasing workload are identical. This is an advantage for the controller design of oxygen uptake regulation for moderate treadmill exercises. As triaxial accelerometers are employed to synchronize body movements with oxygen uptake, time delay of the ARX model is well explored based on synchronized data. Our results support the opinion that time delay of the oxygen uptake dynamic can be ignored. Oxygen uptake of steady state experiments are measured by mixing chamber based respiratory measurement system. A RBF kernel SVM model is achieved based on the reliable steady state data. We believe that the proposed model may be useful in the prediction and regulation of oxygen uptake during treadmill exercises.

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