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Modelling and Control for Heart Rate Regulation during Treadmill Exercise

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Abstract— **This paper proposes a novel integrated approach for the identification and control of Hammerstein systems to achieve desired heart rate tracking performance for an automated treadmill system. The pseudo-random binary sequence input is employed to decouple the identification of dynamic linear part from static nonlinearity. The powerful ϵ-insensitivity Support Vector Regression is adopted to obtain sparse representations of the inversion of static nonlinearity in order to obtain an approximated linear model of the Hammerstein system. An H_∞ controller is designed for the approximated linear model to achieve robust tracking performance. This new approach is applied to the design of a computer-controlled treadmill system for the regulation of heart rate during treadmill exercise. Minimizing deviations of heart rate from a preset profile is achieved by controlling the speed of the treadmill. Both conventional Proportional- Integral- Derivative (PID) control and the proposed approaches have been employed for the controller design. The proposed algorithm achieves much better heart rate tracking performance.**

Index Terms— **heart rate control, Hammerstein model, identification, Support Vector Regression, robust control.**

I. Introduction

AUTOMATED exercise testing systems have become increasingly important in sport training, medical diagnosis, rehabilitation and analysis of cardio respiratory kinetics [1] [2]. These systems can fully implement programmed exercise and training protocols to achieve desired exercising and testing results. The major aim of this study is to develop a computer controlled treadmill system, which can control the heart rate of the subject according to a preset heart rate profile. Some commercial treadmills are already available which offer heart rate control. However, these normally use very simple control strategies, their control performance is poor and they have no mechanism for setting a desired heart rate profile. In this paper we design a treadmill exercise system which can automatically control the treadmill speed to accurately track a desired preset heart rate profile.

PID control is the most popular control algorithms in industry due to its simplicity in structure and ease of tuning. However, this study shows the acceptable heart rate tracking performance cannot be achieved by PID controller because of highly nonlinear behavior of the controlled system. To achieve better tracking performance a new model based robust control approach is developed, which successfully compensated the nonlinearity by using a Hammerstein model [3]. This approach includes two integrated parts: the identification of Hammerstein model and model based robust control. The identification of Hammerstein model is a very active research topic [4]. Recently, Goethals et al [5] presented a novel overparametrization identification approach for Hammerstein systems. The most distinguishing part of that approach is the utility of a powerful machine learning method, Least Square Support Vector Machine (LS-SVM). In this paper, we apply the SVM approach [6] [7] combined with the stochastic method [4] to identify physiological processes. There are at least two aspects which are different with respect to paper [5]:

The stochastic method [4] is employed in preference to the over parameterization method [8]. As discussed in [4], the error of the identification of Hammerstein model is not only from linear and nonlinear parts themselves but also from the coupling between them. The pseudo-random binary sequences (PRBS) are applied to decouple the identification of the two parts as suggested in [4].

Another main difference is the usage of ϵ-insensitivity SVM [9] instead of LS-SVM [10]. Both LS-SVM and ϵ-insensitivity SVM have the merits of SVM approaches. However, the loss function used by ϵ-insensitivity SVM, only penalizes errors greater than a threshold ϵ. This leads to a sparse representation of the decision rule giving significant algorithmic and representation advantages [9]. On the other hand, the ridge regression (ϵ = 0) used by LS-SVM typically causes the loss of sparseness representation.

Most existing control methods for Hammerstein systems are based on direct inversion of the static nonlinearity combined with existing linear control approaches [11]. These methods consider control and identification separately. They identify the Hammerstein model first and then invert the static nonlinearity part to obtain an approximated linear model. There are mainly two disadvantages of these approaches. Firstly, if the identified nonlinearity $\hat{f}(\cdot)$ has input multiplicity then the inversion is unachievable. Secondly, even when the inversion exists uniquely it is often hard to achieve an explicit analytic expression.

Based on ϵ-insensitivity SVR, the inversion of static nonlinearity rather than the nonlinearity is identified directly. Then, the robust H_∞ control is designed for the approximated linear model to achieve robust tracking performance [12] [13]. It should be noted that the inversion of the static nonlinearity cannot be achieved directly by using the method

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proposed in [5].

The proposed identification and control approach was applied for the automated heart rate regulation system design, and excellent tracking results are achieved. This is also the first report of SVM application in the area of cardiovascular system identification and control.

The paper is organized as follows. The details of SVM based identification and control approach is given in Section 2. Section 3 presents the application of the proposed approach for the regulation of heart rate of treadmill exerciser.

II. SVM BASED IDENTIFICATION AND CONTROL APPROACH

In this study, as assumed in most stochastic method (a gray box procedure), the steady-state gain of the linear dynamic model is constrained to be unity, and the steady-state characteristic of the overall model is determined by the static nonlinearity. As mentioned in the introduction, the linear dynamic identification of Hammerstein models can be decoupled from that of nonlinear parts by using pseudo-random binary sequences (PRBS) [4]. Because the PRBS inputs do not excite the nonlinearity sufficiently, to identify the nonlinear part or its inversion, steady state experiments should be performed.

A. Modeling the inversion of static nonlinearity by using SVR

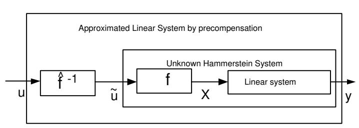

Fig. 1. The precompensated Hammerstein system

To transfer a Hammerstein system to a linear system, a pre-compensator as shown in Fig. 1 can be applied as in [11].

For the identification of the inversion of nonlinearity, the so called ϵ-insensitivity SVR regression will be employed. The formulation of SVM embodies the structure of risk minimization principle [6]. Support Vector Machine based regression (SVR) applies the kernel methods implicitly to transform data into a feature space (this is known as kernel trick), and uses linear regression to get a non-linear function approximation in the feature space. There are a number of kernel functions which have been found to provide good generalization capabilities. Here we give the polynomials and RBF kernel functions as follows:
RBF kernel: $k(u,u') = exp(-\frac{\|u-u'\|^2}{2\sigma^2})$,
Polynomial kernel: $k(u,u') = ((u \cdot u') + b)^d$.

Details about SVR, such as the selection of radius ϵ of the tube, kernel function, and the regularization constant C, can be found in [9], [14] and [15].

B. Identification of linear dynamic part

In [4], Bai showed that the identification of linear part of a Hammerstein model can be decoupled from nonlinear part with the help of the PRBS input. The reason is that any static nonlinearity can be exactly characterized by a linear function, when driven by PRBS inputs which have a binary nature.

When PRBS input is employed, as shown in equation (2.3) of [4], the identification of a Hammerstein model can be simplified as a linear identification problem. Any linear identification approach (parametric or non-parametric) can be applied. The parametric approach as suggested in [4] is adopted in this study.

C. H_∞ based tracking controller design

After the pre-compensator is employed, the Hammerstein system can be treated as a linear dynamic system. However, this approximated linear system often has modeling errors. In this study, the input disturbances and additive model uncertainty are considered as modeling errors. Therefore, an H_∞ controller is employed for the pre-compensated Hammerstein system to achieve desired tracking performance under input disturbance and additive model uncertainty. This controller design problem can be formulated as a mixed sensitivity problem [16] [17] [18]. The diagonal output weighting functions are selected to achieve desired tracking performance and robust stability.

III. HEART RATE REGULATION APPLICATION

The major aim of this section is to apply the proposed identification and control approach to develop a computer controlled treadmill system, which can automatically control the treadmill speed to accurately track a desired preset heart rate profile.

A. Experiment equipments

The treadmill used in the system is the Powerjog "G" Series fully motorized medical grade treadmill manufactured by Sport Engineering Limited, England. Control of the treadmill can be achieved through an RS232 serial port. The treadmill can receive commands from the computer controller via this link, and obeys such commands without supervision. The measurement of heart rate in the designed system is implemented using a wireless Polar system. However, even in the absence of external interference the heart rate can vary substantially over time under the influence of various internal or external factors. Therefore, an improved exponential weighted moving average filter together with a simple outlier detection algorithm [2] is adopted for the estimation of the heart rate.

B. Nonlinearity modeling by using Support Vector Regression

In order to identify the nonlinear relationship, steady state experiments are performed and recorded. Six young healthy male subjects volunteered to participate in the study. Their physical characteristics are presented in Table I.

	Mean	SD	Range
Age (yr)	31.61	5.78	23-37
Height (cm)	176.41	5.48	169-184
Body mass (kg)	74.31	9.35	60-85

TABLE I
SUBJECT CHARACTERISTICS (N=6)

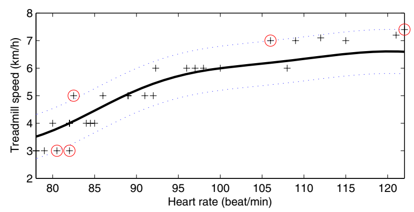

Fig. 2. Inversion of nonlinearity modeling by using ϵ-insensitivity SVR

All experiments were conducted in the afternoon, and the subjects were permitted to have a light meal one hour before measurements were recorded. Initially, the subjects were asked to walk for about 10 minutes on the treadmill to familiarize themselves with the experiment. The subjects were then requested to walk at five levels of different speeds (3 km/h, 4 km/h, 5 km/h, 6 km/h and a subject specific maximum walking speed, typically 7km/hour). Each level took a total period of 5 minutes, and was followed by a 10-minute resting period. Finally, in order to identify the linear dynamic part of the Hammerstein system, subjects were also requested to walk on the treadmill under a PRBS input as shown in Figure 3. Throughout the experiments heart rates were recorded.

In this paper, the ϵ-insensitivity SVR regression method is applied to modeling the nonlinear relationship. The regression results are summarized in Table II and Fig. 2.

Kernel	Parameter	Regularization Constant C
RBF	σ=20.2	5
ϵ-insensitivity	Support vectors number	RMS error
0.8 km/h	5 (16.7 %)	0.5 km/h

TABLE II
DETAILS ABOUT THE ESTIMATION OF THE INVERSION OF NONLINEARITY BY SVR

C. Linear dynamic part modeling of heart rate regulation during treadmill walking

From physiological analysis of cardiovascular systems [19], there are two transitory components take effect for heart rate variation: at the beginning of exercise the rapid component of vagal inhibition and then a much slower acting complex of sympathetic effects. For the dynamic modeling of heart rate variation during exercise, some complicated model structures are proposed based on physiological analysis. However, from a control application point of view, these models are too complicated and often lead to poor determinability of parameters and thus poor control performance. Hajek et al [20] proposed an efficient model as follows:

$$Y(s) = [\frac{K_1 T_i s}{(T_1 s+1)(T_i s+1)} \quad \frac{K_{ref}}{T_i s+1}] \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}. \quad (1)$$

In model (1), $Y(s)$ is the Laplace form of the heart rate variation $y(t)$, and input $u_2(t)$ ($U_2(s)$ is its Laplace form) is the workload. Input $u_1(t)$ ($U_1(s)$ is its Laplace form) is constant 1 (when $u_2(t) > 0$), zero (when $u_2(t) = 0$). As $u_2(t)$ ($50 - 125$ Watt) is much bigger than $u_1(t)$, the effects of $u_1(t)$ is neglected but treated as model uncertainty in order to reduce design complexity. Therefore, a first order model (with input disturbances) to model the heart variation during exercise is a good choice from physiological analysis. Based on the PRBS input experiments data (see Figure 3), the structure is determined by using the Matlab Identification Toolbox. Two popular model selections criteria Akaike information criterion (AIC) and minimum description length (MDL) were used to select the first order ARX model with time delay as the best model structure for linear dynamics. The identified linear model is given as follows:

$$y(k) = 0.648y(k-1) + 0.352u(k-3) + e(k), \quad (2)$$

with sampling period $T_s = 15$ seconds.

D. Robust tracking controller design

The identified model inevitably has modeling errors. In this study, the modeling errors are considered in two forms: input disturbance and model uncertainty. To achieve robust tracking under input disturbances and model uncertainty, a mixed sensitivity H_∞ controller is designed for the identified model [17]. As the design of discrete time mixed sensitivity H_∞ controllers is not as mature as for continuous time systems, the design of the weighting function of the mixed sensitivity H_∞ problem is implemented by using its continuous counterpart. Specifically, we convert the discrete time model (2) into a continuous model. Then, we augment the system with continuous sensitivity weighting functions. This augmented continuous model is then converted to its discrete time form. Finally, a mixed sensitivity H_∞ controller can be designed for this discrete time model:

$$C(z) = \frac{0.3487 - 0.6466z - 0.2899z^2 + 0.6279z^3}{-0.3779 + 1.174z - 1.79z^2 + z^3}. \quad (3)$$

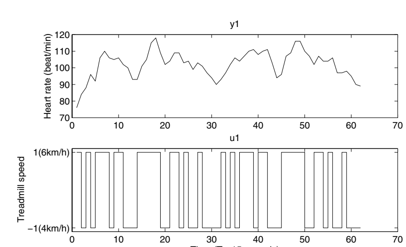

Fig. 3. Two periods of 31 bits PRBS inputs and its corresponding output

E. Heart rate regulation results

The typical step responses of heart rate control by using PID and the proposed control approach are compared in Fig. 4. It can be seen that the proposed controller achieved much better performance than conventional PID control. The main reason is the compensation of nonlinearity by using a Hammerstein model for the heart rate variations.

Finally, the proposed controller was used to track a preset desired heart rate profile. The profile includes three stages: 5 minutes increasing (from resting heart rate to 100 beats/min), 15 minutes holding (110 beats/min), and 5 minutes decreasing (from 110 beats/min to normal.). This is comparable to heart rate profiles recommended for optimal low level aerobic training at about 60% of maximum effort. Figure 5 demonstrates that quite satisfactory tracking of the desired heart rate profile was achieved. From the shape of treadmill speed shown in Figure 5, it is also observed that the treadmill speed is decreased in a slightly different rate for different subjects whilst the heart rate is maintained at around 110 bpm. These data suggest the hypothesis that fit subjects will have a low rate of decrease of treadmill speed. This hypothesis is being further tested in our laboratory, using the new designed computer controlled treadmill.

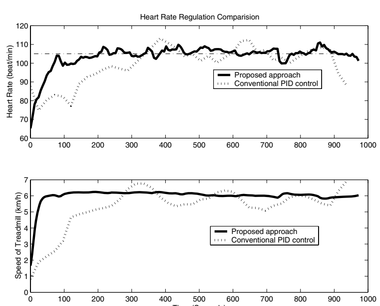

Fig. 4. Typical step responses comparison

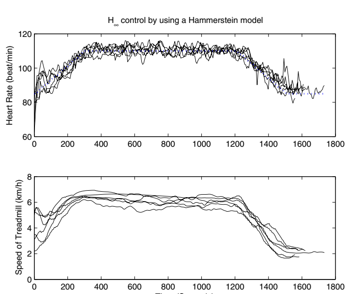

Fig. 5. Heart rate tracking results for all subjects

IV. Conclusion

In this paper, an integrated modeling and control approach is developed for Hammerstein systems as applied to the cardiovascular system response to exercise. The stochastic method is adopted to decouple the identification of linear and nonlinear parts. Powerful ϵ-insensitivity Support Vector Regression is employed to identify the inversion of input nonlinearity in order to transfer Hammerstein systems to linear systems. A robust H_∞ tracking controller is then designed for the converted Hammerstein system. The approach is successfully applied to a heart rate regulation system for exercise on a motorized treadmill. This is the first report of SVM application in the cardiovascular system identification and control practice, and achieves excellent results. It should be mentioned that because of the sparse representation [9] of the static nonlinearity obtained by using ϵ-insensitivity SVR, the implementation complexity is greatly reduced. We believe that the ability to track a predetermined heart rate profile may be useful in cardiac rehabilitation programs or for safer exercise for individuals at risk.

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