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Matching a Wavelet to ECG Signal

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Abstract— **In this paper we develop an approach to synthesize a wavelet that matches the ECG signal. Matching a wavelet to a signal of interest has potential advantages in extracting signal features with greater accuracy, particularly when the signal is contaminated with noise. The approach that we have taken is based on the theoretical work done by Chapa and Rao [10]. We have applied their technique to a noise-free ECG signal representing one cardiac cycle. Results indicate that a matched wavelet, that was able to capture the broad ECG features, could be obtained. Such a wavelet could be used to extract ECG features such as QRS complexes and P&T waves with greater accuracy.**

I. INTRODUCTION

Wavelet transforms provide an approach to multi-resolution analysis of a signal and this technique has been used to identify the ECG signal features such as P-wave, QRS complex and T-wave [1]. In reality, the output produced by the wavelet transform is similar to those of matched filters [2, 3]. In such applications, there is a need for the mother wavelet to resemble the ECG signal particularly when decomposing the signal in the presence of noise. A mother wavelet that matches the signal of interest would produce a sharper peak in time-scale space thus enhancing the ability to better detect signal features. A biorthogonal wavelet has been commonly used to produce the best match for the QRS complex of the ECG [1]. Its success could primarily be due to the fact that the biorthogonal wavelet is very close in shape to the QRS complex. However, the same wavelet does not match the P and T waves with acceptable accuracy [1]. Mallat and Zheng [3] pointed out that a single wavelet basis function is not flexible enough to represent a complicated non-stationary signal such as the ECG. For this reason, techniques have been developed to find orthonormal wavelet bases with compact support [4]-[9]. In these techniques, a dictionary of pre-defined scaling functions is made available to be used in the matching process. The matching algorithm selects the suitable scaling function from the dictionary that provides the best match for the signal of interest. This selection process of the scaling function gives rise to optimal matching for the lower frequency band of the signal. However, using a dictionary of wavelets to optimize the match is influenced by the contents of the dictionary. The dictionary of pre-defined functions might not include functions to compactly represent the signal of interest, e.g. the ECG signal. Also, representing different segments by different functions does not optimally maintain the temporal structure of the signal. To address this problem, Chapa and Rao [10-11] proposed a generalized technique to design a wavelet such that a single wavelet could provide the best match for the signal of interest. In this paper we describe how such a technique could be applied to generate a mother wavelet that matches the ECG signal.

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II. METHODS

Chapa and Rao's algorithm [10] is based on multi-resolution analysis (MRA) to develop an orthonormal wavelet that matches a signal of interest. The orthonormal MRA decomposes a signal, f(x), into a series of detail functions, W_j, and a residual low resolution approximate function, V_j. That is, f(x) is projected onto W_j and V_j, where $V_{j-1}=V_j+W_j$ (note that summation is a vector sum and not an arithmetic sum, where V_j and W_j are orthonormal). The recursive projection of f(x) onto V_j and W_j produces the detail functions $g_j(x)$ and $f_J(x)$, such that,

$$f(x) = f_J(x) + \sum_{j=1}^{J} g_j(x) \qquad (1)$$

The orthonormal bases of W_j and V_j are given by the wavelets $\psi_{j,k}(x)$ and scaling function $\phi_{j,k}(x)$ where,

$$\begin{aligned} &\int \psi(x) = 0 \rightleftarrows \Psi(0) = 0 \\ &\int \phi(x) = 1 \rightleftarrows \Phi(0) = 1 \end{aligned} \qquad (2)$$

and $\Psi(\omega)$ and $\Phi(\omega)$ are the Fourier transform of $\psi_{j,k}(x)$ and $\phi_{j,k}(x)$ respectively.

To obtain a single wavelet that could provide the best match, the signal of interest is projected onto the different orthonormal wavelets. The projection coefficients, d_k^j, in the time and frequency domains are given by,

$$d_k^j = \langle f(x), \psi_{j,k} \rangle = \langle F(\omega), \psi_{j,k}(2^j \omega) \rangle \quad (3)$$

The matching process, in itself, is carried out in two steps. In the first step the spectral amplitudes of the ECG and the wavelet are matched and in the second step their group delays are matched. The required conditions to obtain orthonormal bases functions are independently applied for each step. Though the independent application of the orthonormality conditions on amplitude and phases separately is sub-optimal, this approach was able to generate wavelets that closely match a desired signal [11].

The necessary and sufficient condition to guarantee that the spectrum amplitude of the scaling function, $|\Phi(\omega)|$, generated from the wavelets $\Psi(\omega)$ satisfies the orthonormality condition on phi(x) is orthonormal is given by equation (4) [11].

$$\sum_{m=-\infty}^{\infty} \sum_{p=0}^{l} Y\left(2^{M-p}(n+2^{l+1}m)\right) = 1$$
$$\text{where} \quad Y(k) = |\Psi(k\Delta\omega)|^2, \quad \Delta\omega = 2\pi / 2^M \quad (4)$$
$$2^M / 3 < \left|2^{M-p}(n+2^{l+1}m)\right| < 2^{M+2}/3,$$
$$l = \{0, 1, ..., M\}$$

where Y(k) is the squared amplitude of the wavelet spectrum, and $\Delta\omega$ is the discrete sample spacing of the spectrum. Also, the spectrum amplitude in equation 4 is constrained to satisfy the recursive characteristic of the scaling function given by,

$$\phi(x) = 2\sum_{k=-\infty}^{\infty} h_k \phi(2x-k) \quad (5)$$

A closed-form solution to the infinite sum in equation 4 was achieved by limiting the band of the wavelet and scale function. Also, the recursive characteristic limited the frequency band of the scaling function, $\Phi(\omega)$, to $|\omega|<4\pi/3$, and the frequency band of the wavelet $\Psi(\omega)$ to $2\pi/3<|\omega|<8\pi/3$ [10]. Because the wavelets being designed are assumed to be real the magnitude of the wavelet spectrum is even, $|\Psi(\omega)|= |\Psi(-\omega)|$. For this reason only the spectra for positive frequency indices, k, in the pass band are matched. The condition for k>0 generates a set of linear equality constraints on Y(k). These constraints are used to minimize the error function, E

$$E = \frac{(W - aY)^T (W - aY)}{W^T W}$$
$$\text{where} \quad (6)$$
$$W = \left\{|F(k\Delta\omega)|^2 ; k = \lceil 2^M/3 \rceil, ..., \lfloor 2^{M+2}/3 \rfloor\right\}$$
$$Y = \left\{|\Psi(k\Delta\omega)|^2 ; k = \lceil 2^M/3 \rceil, ..., \lfloor 2^{M+2}/3 \rfloor\right\}$$

The minimization of the above error function optimizes the match between the wavelet and signal amplitude spectra in addition to satisfying the orthonormality requirement.

Similar to the constraints on the amplitude spectrum of the wavelet function, specific constraints are placed on the phase spectrum of the matched wavelet as well. To establish these constraints, an expression of the group delay of $\Psi(\omega)$ in terms of the group delay of the scaling function, $\Phi(\omega)$ is developed. The scaling and wavelet functions are linear summations of the basis elements of V_{-1}, where the coefficients are given by the sequences h_k and g_k. Also, the coefficients g_k can be expressed in terms of h_k [13]. Therefore, the group delay of $\Psi(\omega)$ can be expressed in terms of the group delay of the Fourier transform of h_k ($H(\omega)$) denoted as $\lambda(\omega)$. The matching process of the group delays of the scaling and wavelet functions are subject to the same orthonormality constraints used to match the amplitude spectra [11]. In addition, the group delay has to satisfy periodicity constraints, specifically, the group delays of the scaling and wavelet functions must be even and must be 2π-periodic [12]. The problem of finding the matched group delays is solved by expanding a single period of $\lambda(\omega)$ ($\lambda_T(\omega)$) to a polynomial of order R.

$$\lambda_T(\omega) = \sum_{r=0}^{R/2} c_r \omega^{2r} \Pi(\frac{\omega}{2\pi})$$
$$\text{where} \quad \Pi(\omega) = \begin{cases} 1 & -0.5 \le \omega \le 0.5 \\ 0 & otherwise \end{cases} \quad (7)$$

$$\lambda(\omega) = \sum_{k=-\infty}^{\infty} \lambda_T(\omega - 2\pi k)$$
$$= \sum_{k=-\infty}^{\infty} \sum_{r=0}^{R/2} c_r (\omega - 2\pi k)^{2r} \Pi(\frac{\omega - 2\pi k}{2\pi}) \quad (8)$$

The group delays Λ_Φ and Λ_Ψ of $\psi(\omega)$ and $\Phi(\omega)$, respectively, can be expressed in terms of $\lambda(\omega)$ through the following matrix equations [11]

$$\Lambda_\Phi = D_\Phi c$$
$$\Lambda_\Psi = -\Delta\omega/2 + D_\Psi c$$
$$\Gamma_\Psi = D_\Psi c$$
$$\text{where} \quad (9)$$
$$D_\Phi = \sum_{m=1}^{\infty} 2^{-m} B_{q/2^m}$$
$$D_\Psi = -\frac{1}{2} B_{(q+T)/2} + \sum_{m=1}^{\infty} 2^{-m} B_{(q+T)/2^m}$$
$$B_{n,r} = \sum_{k=-P/2}^{P/2-1} (n - kT)^{2r} \Pi(\frac{n-kT}{T})$$

The matching process of the group delays of the wavelets to that of the signal of interest is carried out by minimizing the squared error between their phases. The minimization process is constrained by the orthonormal conditions to provide estimated values for the coefficients, c, that define the polynomial expression of the group delays [11]

$$\hat{c} = (\bar{D}_\Psi^T \bar{D}_\Psi)^{-1}(\bar{D}_\Psi^T \bar{\Gamma}_F) \quad (10)$$

The best estimate of 'c' can be used to find values of Λ_Φ and Λ_Ψ

$$\lambda = B\hat{c}$$
$$\Lambda_\Psi = (D_\Psi \hat{c} - \overline{D_\Psi \hat{c}}) - \frac{\Delta\omega}{2} \quad (11)$$
$$\Lambda_\Phi = (D_\Phi \hat{c} - \overline{D_\Phi \hat{c}})$$

III. RESULTS

A single cardiac cycle of the ECG signal sampled at 1024 Hz was extracted from an artifact-free recording of lead II electrocardiogram. The above described technique implemented in Matlab (Mathwork Inc, Natick, MA) was used to obtain a matched wavelet. The matched wavelet and the ECG signal are shown in Figure 1. Though an exact match is not obtained, the matched wavelet captured the main features of the ECG; the P-wave, the QRS complex, as well as the T-wave.

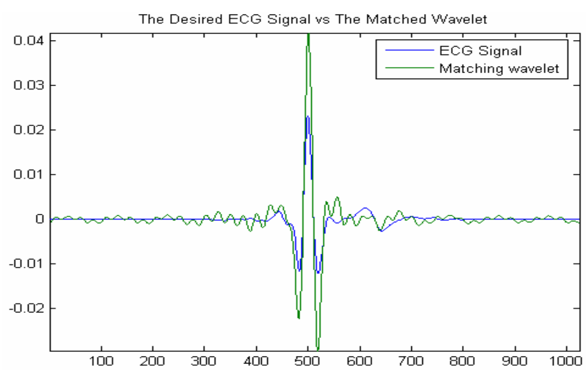

Figure 1 - The ECG signal verses the matched wavelet

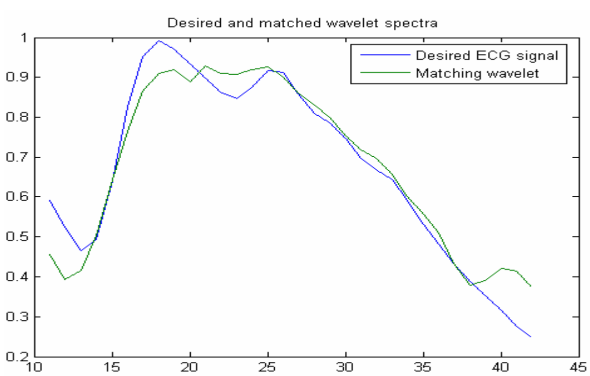

Figure 2 - Amplitude spectra of the ECG signal and the matched wavelet

The comparison of the amplitude of the signal spectrum to that of the matched wavelet is displayed in Figure 2. It is evident that the matching technique achieved a close match of the amplitude spectra with a squared error of 0.0075.

The group delay of the ECG signal is displayed against that of the matched wavelet in Figure 3. Unlike the amplitude spectrum, the group delay of the matched wavelet did not closely match in the low frequency band of the ECG. This is primarily due to the fact that the ECG signal had to be band limited to satisfy the required orthonormality constraints.

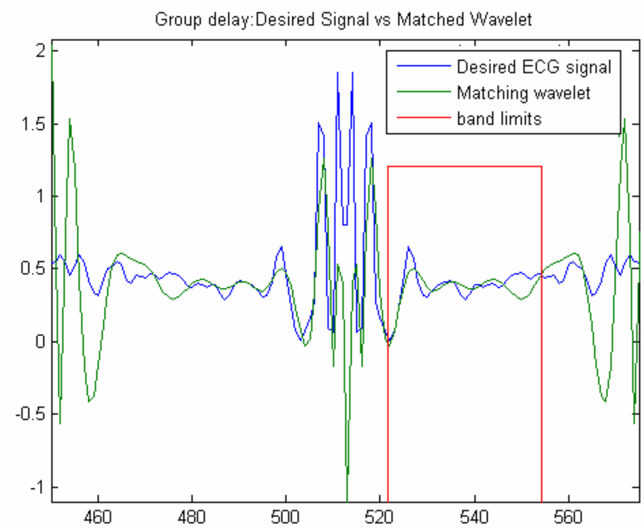

Figure 3 - Group delay of the ECG signal and the matched wavelet.

IV. SUMMARY

The technique described in the paper for wavelet synthesis provides a mechanism to create a specialized orthonormal basis that is suitable to match a signal of interest. The main features of the ECG signal can be captured with a matching wavelet that preserves the temporal sequence of the features. However, to achieve an exact wavelet match to the signal of interest, the signal has to be orthonormal. Meyer's wavelet is an example of an orthonormal signal that can have an exact wavelet match using the proposed algorithm [11]. The ECG signal is not orthonormal and hence an exact match cannot be expected. However, an exact match of the ECG signal might not be necessary because the wavelet itself will be used to decompose the signal into different scales. For a matched wavelet, we expect that the signal energy can be captured in a single scale or two.

It can be observed that the matched wavelet includes ripples at the baseline of the ECG signal. These ripples are a result of using hard cutoff edges of rect function to obtain the necessary passband. This problem can be solved by using a smooth function such as cubic or gaussian. Also, it's worth noting that there are precision errors from calculating matrix inversions as part of the matching algorithm.

The matched wavelet needs to be evaluated to gauge its performance in extracting ECG features and detecting artifacts in noisy ECG signals acquired from real environment.

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