

A Nonlinear System Model of Isometric Force

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Abstract— The analysis of isometric force may provide early detection of certain types of neuropathology such as Parkinson's disease. Our long term goal is to determine if there are detectable differences between model parameters of healthy and unhealthy individuals. In this study we used system identification techniques to estimate the parameters of dynamic system models of the isometric force exerted by the index finger and focused on a single category of subjects, healthy young adults. The experiments involved subjects exerting isometric force over a range from 5% to 95% of maximal voluntary contraction. The coefficients of the differential equation models depended on the target force level. This finding suggests that a nonlinear dynamic system model provides the best fit for isometric force experiments.

I. INTRODUCTION

The purpose of this study was to model the isometric force that is applied by the index finger. The data were collected from young healthy adults to serve as a standard dataset for comparisons across age and across various neuropathologies. The transient portions of the datasets were used to parameterize models, and the values of the estimated parameters were compared to determine if a single linear time-invariant (LTI) model was applicable across the entire isometric force range of an individual. A secondary goal was to determine if a single LTI model was applicable across an entire category of subjects, in this case healthy young adults.

Previous studies have shown that two salient modes are present in the power spectral density (PSD) of isometric force recordings. Both of these sub-regions of the PSD are associated with physiological tremor, and deviations can be used to discriminate between physiological and pathological tremor, there by providing an early detection of neurological disorders such as

Parkinson's tremor [1]. Our ultimate goal is to develop a diagnostic tool that can be applied in a physician's office for early detection and more effective treatment.

There are two putative sources of the two physiological tremor distributions. The first is a neuronal tremor that falls consistently in the range of 8-12 Hz; this form of tremor is believed to originate in the motor cortex and to be resistant to change [2]. The second form of tremor, termed mechanical tremor, is a form of action tremor that is associated with the stretch reflex; this form has also been referred to as isometric tremor [2]. Isometric tremor is often simulated by an oscillator that models the feedback loop formed by stretch receptors, spinal cord, and motoneurons innervating agonistic and antagonistic muscles.

Previous studies of isometric force experiments on the index finger suggest the possibility of discriminating between healthy subjects and those afflicted with Parkinson's disease [3]. The change in dynamical complexity in the steady-state region of the isometric force response has been analyzed using the approximate entropy statistic [4, 5]. In addition, piece-wise linear stochastic maps have been used to model the isometric force response of the index finger [6, 7].

In this study we examined isometric force responses from the linear control system perspective [8]. Specifically, system identification was employed to estimate the parameters of a linear model [9] and the resulting isometric force response models were found to be nonlinear functions of the target force.

II. METHODS

The subjects were 3 undergraduate students who provided informed consent prior to participation. The task was to produce a constant level of isometric force in index finger adduction (medial

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lateral motion of the middle finger and arm were restricted) so that the force output on a computer screen matched the target force level, i.e., 5, 15, 25, 35, 45, 55, 65, 75, 85, and 95% maximal voluntary contraction (MVC). The task-related normal forces and tangential force were collected with a 3-dimensional load cell. The trial length was 15 s. The force levels were presented to each subject in random order, and each subject completed five repetitions at each force level. The first 2.5 s of a trial defined the transient response. The sample rate for data collection was 100 Hz (i.e., $T = 0.01$ s).

In order to study the feedback control systems involved in the isometric force response, we developed models of the transient response. Using system identification techniques, a model was parameterized for each trial of the target force level.

Fig. 1 shows the plots of five repetitions by one individual at one force level. The five trials show a range of responses consonant with a second-order LTI model for the deterministic step response of the system [8, 10].

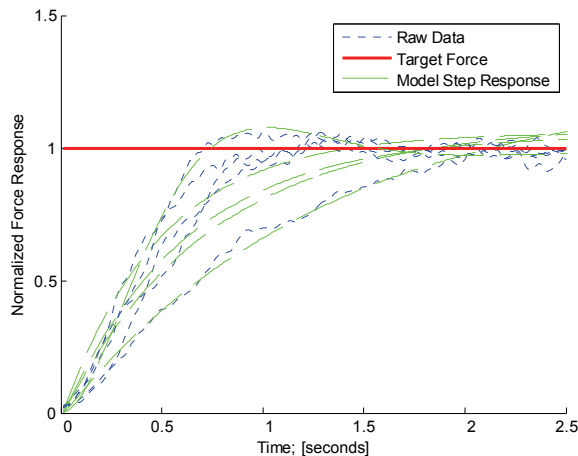


Figure 1 – Plot of five repetitions by one subject at one level.

The stochastic component of the transient response was modeled as an autoregressive moving average (ARMA) random process [11]. The ARMA (2, 2) model family was determined to be the best low-order model of the disturbance term [9]. A second-order system was determined to be adequate to model the range of step responses such as the over-shoot observed in Fig.

1, while not over-fitting the model to the data. The Matlab™ System Identification toolbox was used to estimate the parameter values. For each trial, the least-squared error system identification algorithm was used as the estimation the coefficients of an ARMAX (2, 1, 2) model.

The general form of the parametric model that was fitted to the raw data is described in Equation 1.

$$A(q)y(t) = B(q)u(t) + C(q)e(t)$$

Where :

$$A(q) = a_0(t)q^0 + a_1(t)q^1 + a_2(t)q^2 \quad (1)$$

$$B(q) = b_0(t)q^0$$

$$C(q) = c_0(t)q^0 + c_1(t)q^1 + c_2(t)q^2$$

The symbol q^i represents the i^{th} order backward shift operator [i.e., $q^2y(t) = y(t - 2T)$]. The rational function $B(q)/A(q)$ describes the system's transfer function. The ARMA disturbance term is modeled by the rational function $C(q)/A(q)$ which shapes the PSD of the zero-mean Gaussian noise source $e(t)$.

III. RESULTS

The MVC and corresponding target force levels differed across subjects. To standardize the responses across individuals we used an integer index corresponding to increasing target force level. This index was termed the condition index and is used as the independent variable on the following figures.

Fig. 2 plots the values of the first AR factor, a_1 , vs. the condition index. Each point on the plot represents an average of five trials along with the standard error of the mean. The plots of the three subjects show a clear decrease in the estimated value of the AR coefficient with increasing force. Statistical analysis confirmed that the trend was significant with a slope of -0.023839. The line plotted on Fig. 2 is the average of the three lines following a linear regression of the coefficient onto the condition indices.

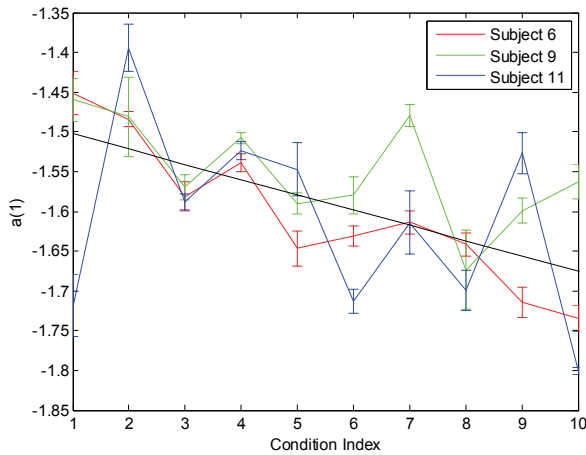


Figure 2 – Plots of the decreasing value of the first AR coefficient, a_1 , with increasing target force level. Each trace is the average of five repetitions of the isometric force experiment by 1 of the 3 subjects. The error bars indicate the standard error of the mean. The solid line is the average linear regression.

As expected, the values of the poles in the complex-frequency domain displayed a corresponding change in value. Fig. 3 is the plot of the change in position of the second (real) pole as the target force increased.

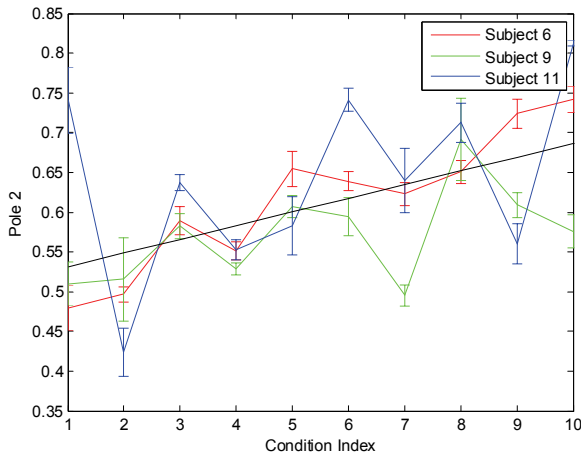


Figure 3 – Plots of the increasing value of one of the complex-frequency domain poles for the three subjects, analogous to Fig. 2. The error bars indicate the standard error of the mean. The solid line is the average linear regression.

Each point on the plot represents an average of five trials along with the standard error of the

mean. Again, the slope was determined to be significant at 0.021696. The black line plotted on Fig. 3 is the average linear regression of the pole values onto the condition indices.

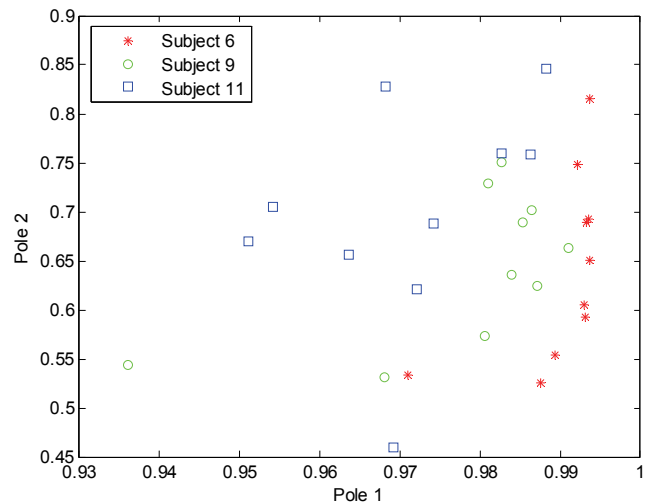


Figure 4 – Scatter plots of the change in pole locations of the transfer functions with increasing target force level. The symbols and line patterns are consistent with the preceding figures.

Fig. 4 is a scatter plot of the locations of the first pole vs. the second pole. The symbols and line styles for the three subjects are consistent with Figs. 2 and 3. The clusterings of the poles on the 2D scatter plots are clearly visible.

IV. DISCUSSION

The first of our two objectives in this study was to determine if a single LTI model could be fitted to the entire range of target force responses. The requirement of a linear system is that it conforms to the principle of superposition [8, 9]. In other words, the coefficients of the difference equations remain constant. If the system is not time invariant these coefficients may vary with time but are otherwise constant. Figs. 2 and 3 clearly illustrate the dependence of the coefficients of isometric force models on the value of the target force. Thus the null hypothesis of a single linear system model across target force range is rejected.

Given the nonlinear nature of the system, each of the coefficients of the input-output model was

replaced by a function of the target force. Equation 2 shows the relation for the characteristic equation of the model.

$$A(q) = q^0 + [m_{a1}x + b_{a1}]q^1 + [m_{a2}x + b_{a2}]q^2 \quad (2)$$

The input variable, x , in Eq. 2 symbolizes the value of the target force. The slopes and y -intercepts of each subject were sufficiently similar to the average values. Table 1 shows the average values of the slope and y -intercept for the two variable denominator terms in Eq. 1

TABLE I
SLOPE AND Y-INTERCEPTS OF THE TWO VARIABLE FACTORS OF SYSTEM CHARACTERISTIC EQUATION.

Coefficients	Slope	Y-Intercept
a_1	-0.023839	-1.5099
a_2	0.0227	0.52297

In the second part of the analysis we were interested in assessing whether a single set of model parameters would be sufficient for a specific group of subjects (e.g. healthy young adults). The larger purpose of the proposed diagnostic system is to detect significant changes in some or all of the models' parameters when comparing healthy individuals and those in the early stage of disease. The data plotted in Fig.4 illustrate that model parameters are subject-dependent. Through the application of pattern recognition techniques, however, it may be possible to discriminate among subjects even within this homogeneous group.

V. CONCLUSION

The data indicated that the isometric force response on the index finger is dependent upon the level of target force. This result is best described by a non-linear dynamic model that was non-linear in the coefficients of the difference equation. Our analysis also showed that using parameters of the model as features; it was possible to discriminate among individuals.

Future work will be directed at evaluating random models in the steady-state region of the recordings. This region provides stationary signals

that are useful for determining models of tremor sources. In addition, pattern recognition algorithms will be developed to provide automatic discrimination among subjects.

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