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A FDM anisotropic formulation for EEG simulation

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Abstract—**Accurate head modeling is required to properly simulate bioelectric phenomena in 3-D as well as to estimate the 3-D bioelectric activity starting from superficial bioelectric measurements and 3-D imaging. Aiming to build an accurate and realistic representation of the volume conductor of the head, also the anisotropy of head tissues should be taken into account. In this paper we describe a new finite-difference method (FDM) formulation which accounts for anisotropy of the various head tissues. Our proposal, being based on FDM, derives the head model directly from patient's specific clinical images. We present here the details of the numerical formulation and the method validation by comparing our numerical proposal and known analytical results using a multi-shell anisotropic head model with skull anisotropy. Furthermore, we analyzed also different numerical grid refinement and EEG source characteristics. The comparison with previously developed FDM methods shows a good performance of the proposed method.**

I. Introduction

NEURAL brain sources produce scalp surface potentials measurable using electroencephalography (EEG). Though the electrical activity of neurons is conveniently modeled as current sources within the brain, it is well known that the estimation of the current source parameters from the EEG is a highly ill-posed problem. This procedure is referred to as the EEG inverse problem solution and generally consists of repeated forward problem solutions, i.e., computing the scalp potentials generated by a known source in the brain solving the differential Poisson's equation, then varying iteratively source parameters until the calculated potential distribution best fits the measured activity. To this end we use the concept of "equivalent source" (e.g., a single current dipole), in conjunction with a specified volume conductor which should be geometrically and electrically similar to subject's head [1]. The volume conductor head model mathematically expresses the relationship between the current source inside the volume and the corresponding surface potentials. Numerical methods have to be used to solve Poisson's equation in realistic head models, constructed out of segmented medical images, as the Boundary Element Method (BEM), the Finite Element Method (FEM) and the Finite Difference Method (FDM) [2].

Despite significant progress in advanced numerical methods to solve the inverse problem over the past two decades, the inaccuracy of the head model itself remains a major source of error [3]–[5]. Studies reported in the literature to assess the accuracy of models for EEG forward/inverse problems are mainly based on the assumption that the tissues have isotropic conductivity values. It is, however, known that the layered skull's structure leads to a conductivity anisotropy in the skull with a ratio of about 1:10 (radially to tangentially to the skull surface) [6]. Neglecting skull's anisotropy in the forward problem solution can lead to spurious errors in EEG source reconstruction [6]. Conductivity anisotropy characterizes also other tissues in the head, e.g., brain white matter. A ratio of about 1:9 (normal to parallel to fibers) has been measured for brain white matter [7].

In the presence of electrical conductivity anisotropy of the physical medium, conductivity is described by a tensor. Under the assumption that the conductivity tensor shares the eigenvectors with the water diffusion tensor of the physical medium [8], measured by Diffusion Tensor Imaging (DT-MRI), the information about tissue conductivity tensor can be extracted from the DT-MRI images and passed to the EEG forward problem formulation. In Boundary Element models the tissue boundaries are formed by discrete triangular elements, with the disadvantage that the surfaces are closed and hence realistic tissue inhomogeneities and anisotropy cannot be taken into account. Such tissue complexities can be incorporated in Finite Element (FE) models, which model the head as an irregular mesh of finite volumetric elements of various shapes and sizes, with a difficulty in generating a mesh of elements with similar tissue characteristics on an irregular grid. We adopt the Finite Difference (FD) approach, according to which the head model is composed of cubic voxels centered on nodes placed on a regular Cartesian grid [9], with a mesh generation simpler than for the FE model. In addition, segmentation of MRI can be eventually avoided since volume elements (voxels) from the image can be mapped directly to the elements of the head model. A FD formulation able to account for conductivity anisotropy as straightforward as possible from imaging information is therefore desirable.

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An original FD problem formulation is proposed in this paper, valid for generally inhomogeneous and anisotropic structures, i.e., characterized by a full conductivity tensor that has non-zero off-diagonal terms and varies with position. The FD formulation is conceived to account for conductivity anisotropy as directly as possible from imaging information, to avoid preliminary pre-processing due to the specific geometry of the adopted mesh as in previously developed problem formulations [10] and to gain a more straightforward problem implementation. To confirm the validity of the new approach, numerical results are compared with an analytical solution [11] developed for EEG forward problem solution for an anisotropic multi-shell spherical head model simulating anisotropy of the skull.

II. THE FD PROBLEM FORMULATION

A. 2-D formulation

The proposed FD formulation for inhomogeneous anisotropic field problems is first derived in the two dimensional (2-D) space for an easy understanding. The FD formulation has similarities to the one proposed and implemented by Saleheen and Kwong in [10] to determine the potential distribution in a canine torso during electrical defibrillation. The method presented in [10] differs from standard FD formulations since voxels are mapped as mesh elements and nodes of the mesh correspond to voxels' vertexes. In the method proposed in this paper, the way the mesh is developed is different and follows standard FD formulations, as it considers the centre of the voxels as the mesh´s nodes and the computational points set in the voxels' center (see Fig. 1), with a one-to-one correspondence between nodes and voxels. Node 0 in Fig. 1 represents a typical node surrounded by eight neighbors.

Problem formulation starts from Laplace's equation:

$$\nabla \cdot \left(\overline{\overline{\sigma}} \nabla u \right) = 0 \tag{1}$$

where $\overline{\overline{\sigma}}$ is the conductivity tensor and u the electric potential. Representing the 2-D tensor in Cartesian coordinates

$$\overline{\overline{\sigma}} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} \tag{2}$$

the Laplacian operator can be expanded in terms of the corresponding derivatives as

$$\sigma_{xx}\frac{\partial^2 u}{\partial x^2} + \sigma_{yy}\frac{\partial^2 u}{\partial y^2} + 2\sigma_{xy}\frac{\partial^2 u}{\partial x \partial y} + \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \right)\frac{\partial u}{\partial x} + \left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} \right)\frac{\partial u}{\partial y} = 0 \tag{3}$$

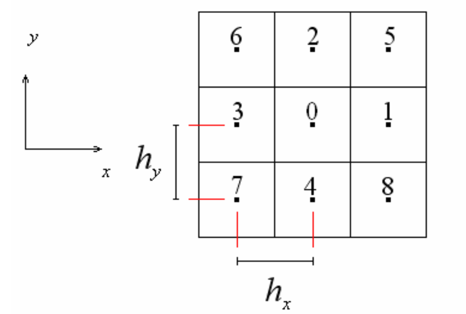

Fig. 1. 2-D discretization in proposed method. FDM computational nodes are set in the center of a tissue element.

Taylor series expansions around node 0 are developed for the products of the conductivities and potentials (σu) in eq. 3 at the neighboring nodes 1-8, where σ is an appropriate entry of conductivity tensors associated with neighboring elements around node 0. Diagonal terms of conductivity tensor are used for nodes lying in principal directions from node 0 while off-diagonal terms are used in diagonal directions. Only terms up to the second order are considered in the series expansions, constituting a system of eight different equations, one for each node. The derivatives $\partial u/\partial y$, $\partial u/\partial x$, $\partial^2 u/\partial x^2$, $\partial^2 u/\partial y^2$ and $\partial^2 u/\partial x \partial y$ at node 0 can then be expressed in term of potential and conductivities at all the nodes (0, …, 8) by solving these equations [10]. Substituting the expressions obtained for derivatives into eq. 3, we obtain the potential at node 0 in terms of the potentials and conductivities at the surrounding nodes, which can be expressed by:

$$\sum_{i=1}^{8} A_i u_i = \left(\sum_{i=1}^{8} A_i \right) u_0 \tag{4}$$

with

$$A_1 = \sigma_{xx(1)}\left(\frac{1}{h_x^2} - \frac{\sigma_x}{\sigma_{xx(0)}} \right), \quad A_2 = \sigma_{xx(2)}\left(\frac{1}{h_y^2} - \frac{\sigma_y}{\sigma_{yy(0)}} \right),$$
$$A_3 = \sigma_{xx(3)}\left(\frac{1}{h_x^2} + \frac{\sigma_x}{\sigma_{xx(0)}} \right), \quad A_4 = \sigma_{yy(4)}\left(\frac{1}{h_x^2} + \frac{\sigma_y}{\sigma_{yy(0)}} \right),$$
$$A_5 = \frac{\sigma_{xy(5)}}{2h_x h_y}, \quad A_6 = -\frac{\sigma_{xy(6)}}{2h_x h_y}, \quad A_7 = \frac{\sigma_{xy(7)}}{2h_x h_y}, \quad A_8 = \frac{\sigma_{xy(8)}}{2h_x h_y}, \tag{5}$$

where

$$\sigma_x = \frac{1}{2h_x^2}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \right)\bigg|_{(0)}, \quad \sigma_y = \frac{1}{2h_y^2}\left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} \right)\bigg|_{(0)}.$$

Indexes (0) - (8) indicate the node at which the derivative of the conductivity is considered, which is associated with a node corresponding to a voxel. Since conductivity is considered to be constant over a voxel, the general term $\partial \sigma_x/\partial x|_{(0)}$ is zero and the terms σ_x and σ_y disappear. It can be noticed that in this formulation the conductivity of the central voxel 0 is not taken into consideration. Furthermore, to avoid neglecting it, the general term $\sigma_{i,j(k)}$ is taken as the average of the term $\sigma_{i,j}$ at node k and node 0, with $i = x, y,\ j = x, y$ and k=1, ..., 8. With this approach a sort of smooth transition of conductivity between the elements of the mesh can be achieved. This approximation guarantees accurate results and it increases speed convergence.

The final 2-D FD formulation is given by:

$$\sum_{i=1}^{8} A_i u_i = \left(\sum_{i=1}^{8} A_i\right) u_0$$

with:

$$A_1 = \frac{1}{2h_x^2}\left(\sigma_{xx}^{[1]} + \sigma_{xx}^{[0]}\right), \qquad A_2 = \frac{1}{2h_y^2}\left(\sigma_{yy}^{[2]} + \sigma_{yy}^{[0]}\right), \tag{6}$$

$$A_3 = \frac{1}{2h_x^2}\left(\sigma_{xx}^{[3]} + \sigma_{xx}^{[0]}\right), \qquad A_4 = \frac{1}{2h_y^2}\left(\sigma_{yy}^{[4]} + \sigma_{yy}^{[0]}\right),$$

$$A_5 = \frac{\sigma_{xy}^{[5]} + \sigma_{xy}^{[0]}}{4h_x h_y}, \qquad A_6 = \frac{\sigma_{xy}^{[6]} + \sigma_{xy}^{[0]}}{4h_x h_y},$$

$$A_7 = \frac{\sigma_{xy}^{[7]} + \sigma_{xy}^{[0]}}{4h_x h_y}, \qquad A_8 = \frac{\sigma_{xy}^{[8]} + \sigma_{xy}^{[0]}}{4h_x h_y}.$$

Indexes [0]–[8] individuate the voxel to which tensor entry refers. This formulation presents a leading error of h^2.

A. 3-D formulation

Problem formulation is then extended to the 3-D case. In conformity with the indexes of Fig. 2, where node 0 is surrounded by 18 neighbors, the final 3-D FD formulation becomes:

$$\sum_{i=1}^{18} A_i u_i = \left(\sum_{i=1}^{18} A_i\right) u_0 \tag{7}$$

with

$$A_1 = \frac{1}{2h_x^2}\left(\sigma_{xx}^{[1]} + \sigma_{xx}^{[0]}\right), \qquad A_2 = \frac{1}{2h_y^2}\left(\sigma_{yy}^{[2]} + \sigma_{yy}^{[0]}\right),$$

$$A_3 = \frac{1}{2h_x^2}\left(\sigma_{xx}^{[3]} + \sigma_{xx}^{[0]}\right), \qquad A_4 = \frac{1}{4h_y^2}\left(\sigma_{yy}^{[4]} + \sigma_{yy}^{[0]}\right), \tag{8}$$

$$A_5 = \frac{\sigma_{xy}^{[5]} + \sigma_{xy}^{[0]}}{4h_x h_y}, \qquad A_6 = \frac{\sigma_{xy}^{[6]} + \sigma_{xy}^{[0]}}{4h_x h_y}$$

$$A_7 = \frac{\sigma_{xy}^{[7]} + \sigma_{xy}^{[0]}}{4h_x h_y}, \qquad A_8 = -\frac{\sigma_{xy}^{[8]} + \sigma_{xy}^{[0]}}{4h_x h_y},$$

$$A_9 = \frac{1}{2h_z^2}\left(\sigma_{zz}^{[9]} + \sigma_{zz}^{[0]}\right), \qquad A_{10} = \frac{1}{2h_z^2}\left(\sigma_{zz}^{[10]} + \sigma_{zz}^{[0]}\right),$$

$$A_{11} = \frac{\sigma_{yz}^{[11]} + \sigma_{yz}^{[0]}}{4h_y h_z}, \qquad A_{12} = -\frac{\sigma_{yz}^{[12]} + \sigma_{yz}^{[0]}}{4h_y h_z},$$

$$A_{13} = \frac{\sigma_{yz}^{[13]} + \sigma_{yz}^{[0]}}{4h_y h_z}, \qquad A_{14} = -\frac{\sigma_{yz}^{[14]} + \sigma_{yz}^{[0]}}{4h_y h_z}, \qquad A_{15} = \frac{\sigma_{xz}^{[15]} + \sigma_{xz}^{[0]}}{4h_x h_z},$$

$$A_{16} = -\frac{\sigma_{xz}^{[16]} + \sigma_{xz}^{[0]}}{4h_x h_z}, \qquad A_{17} = \frac{\sigma_{xz}^{[17]} + \sigma_{xz}^{[0]}}{4h_x h_z}, \qquad A_{18} = -\frac{\sigma_{xz}^{[18]} + \sigma_{xz}^{[0]}}{4h_x h_z}.$$

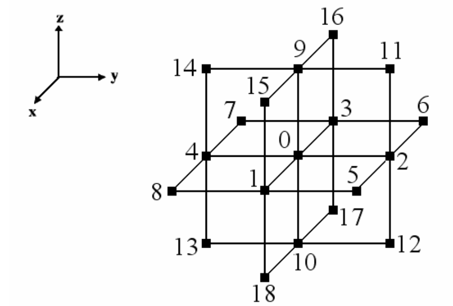

Fig. 2. Indexes used in formulas for central node of the FDM computation and the surrounding neighboring nodes.

II. VALIDATION PROCEDURE OF THE FD PROBLEM FORMULATION

The FD formulations derived in the last section give a nodal equation that expresses the potential at each node as a function of its neighboring nodal conductivity and potential values. All the nodal equations form a large sparse linear algebraic equations system Ax=b, with 19 non-zero entries per row in the 3-D case, which can be solved using standard linear system solution techniques. The bi-conjugate gradient method has been chosen for this work. The FDM solution has been implemented partially in C++ and partially in Matlab. Matrix A, which accounts for head model geometry and conductivity properties, is built up with a C++ routine which generates the system matrix, given the volume discretization mesh. This routine builds up also the system's right hand side according to the position, the direction and the moment of the adopted equivalent dipole source. The linear system solution is obtained calling appropriate Matlab routines.

In order to provide a validation of the proposed anisotropic FD formulation, the EEG forward problem has been solved analyzing a number of test dipole sources in a multi-shell anisotropic concentric-spheres head model, with 4 concentric spheres corresponding to the scalp, the skull, the cerebrospinal fluid (CSF) and the brain compartments, with an anisotropic skull with a radial to tangential conductivity ratio set at 1:10. Compartments' conductivity values and radii have been assigned as shown in Fig. 3. Four

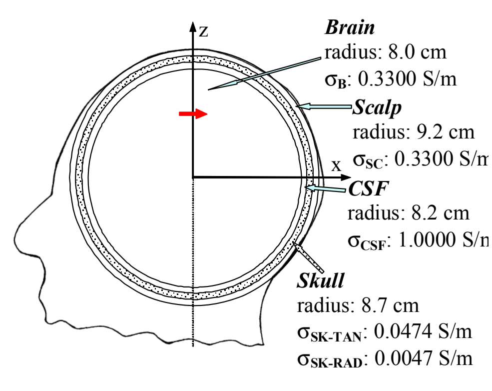

Fig. 3. The 4-shells concentric-spheres head model adopted in the validation procedure.

dipole sources have been considered, placed on the z-axis at 1.5, 3, 4.5 and 6 cm of distance from model center, either tangentially or radially orientated (x- and z-oriented sources, respectively), thus totaling 8 test source conditions. In each test condition, the FDM forward computation of the generated potential has been performed and the numerical solution on the outer model's surface has been compared with the analytical problem solution in the same test condition computed by means of the expressions provided by Zhou and van Oosterom in [11] for a dipole source placed in the innermost sphere of a 4-layer anisotropic model.

The comparison between the FDM and the analytical solution has been performed by means of the correlation coefficient (CC), defined as:

$$CC = \left(\sum_{i=1}^{N} (u_i - \bar{u})(\hat{u}_i - \bar{\hat{u}}) \right)^2 \Big/ \left(\sum_{i=1}^{N} (u_i - \bar{u})^2 \right) \left(\sum_{i=1}^{N} (\hat{u}_i - \bar{\hat{u}})^2 \right) \quad (9)$$

where u_i and $\hat{u}_i$ represent the potential value at node i, respectively in the analytical and in the FDM solution, while $\bar{u}$ and $\bar{\hat{u}}$ represent the mean values. CC gives a quantitative measurement on how the analytical and FDM solutions match each other, approaching to 1 as analytical and numerical solutions get closer. To analyze the FD formulation performance as function of volume conductor's discretization, three different meshes have been tested, with 40^3, 80^3, and 100^3 nodes, leading to corresponding inter-nodal distances of 5, 2.5 and 2 mm respectively.

III. Validation Results

Results of the validation procedure are presented in Tables I and II for the radial and tangential dipole test sources analyzed respectively.

Results of the validation procedure indicate the validity of the proposed FD formulation providing matching evaluation between the solution provided by the FDM and the analytical forward problem solution. The anisotropic model considered in the present study provided valid results ensuring high level of CC in all the analyzed test conditions.

TABLE I

CC values for validation procedure for radial test sources. Source position is given from the center of the spherical model.

	Mesh size		
Source position [cm]	40^3	80^3	100^3
1.5	0.9914	0.9970	0.9985
3.0	0.9645	0.9875	0.9930
4.5	0.9034	0.9696	0.9839
6.0	0.7809	0.9414	0.9684

TABLE II

CC values for validation procedure for tangential sources. Source position is given from the center of the spherical model.

	Mesh size		
Source position [cm]	40^3	80^3	100^3
1.5	0.9952	0.9975	0.9986
3.0	0.9795	0.9895	0.9933
4.5	0.9441	0.9734	0.9847
6.0	0.8678	0.9448	0.9679

From the validation procedure it is also possible to analyze the correlation coefficient's values in relation to model parameters, i.e., number of mesh points and dipole source position. The CC increases as the mesh increases dimension and closer the dipole to the model's centre. Increasing the mesh dimension corresponds to a finer domain discretization and therefore a more accurate modeling. In fact, the error in the FD formulation is proportional to the discretization step h, which decreases as the nodes number increases. It is therefore expectable that the accuracy of the FD solution will increase together with the mesh resolution. Also the dipole source approximation, which is actually represent by two monopoles in the problem formulation, will lead to an error. If the solution is calculated relatively far from the source, the effect of this approximation can be negligible; however, if the dipole is quite close to model surface, where the solution is computed, this can negatively affect the solution. Therefore, since the forward problem solution is calculated over the sphere surface, the results are more accurate with reduced dipole eccentricity. In almost all the situations under investigation the correlation coefficient shows a value high enough (close to 1) to provide a persuasive validation of the method ensuring a valid FD solution of the forward problem. The low resolution model with cortical dipoles produced relatively low correlation. However, this does not originate from the formulation but from the poor representation of skull with coarse 5 mm elements. Moreover, the comparison with the FDM methods presented in [10] confirms the good performance of the method proposed in the present paper [12].

Finally, thanks to the flexibility of our method, we could easily build a realistic head model including information on white matter anisotropy (derived from DT-MRI). The potential distribution due to a dipole source in the brain was calculated using the proposed FD formulation either including or neglecting white matter anisotropy in the model. Fig. 4 shows, in pseudocolor map, the difference of the potential distributions obtained in these two conditions. Such result confirms the importance of being able to account in a simple way for the anisotropy of head tissues in modeling procedures.

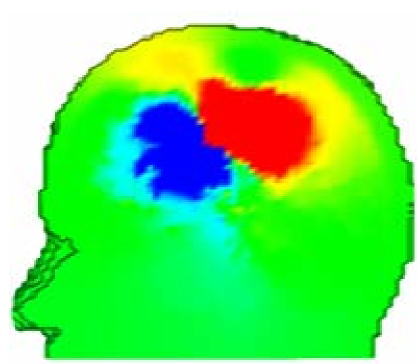

Fig. 4. Difference between potential distribution due to a dipole source in the brain, calculated either including or neglecting white matter anisotropy in the model and plotted with a pseudocolor map.

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