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Modeling the time-varying microstructure of simulated sleep EEG spindles using time-frequency analysis methods

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Abstract— **The time-varying microstructure of sleep spindles may have clinical significance and can be quantified and modeled with a number of techniques. In this paper, sleep spindles were regarded as AM-FM signals modeled by six parameters. The instantaneous envelope (IE) and instantaneous frequency (IF) waveforms were estimated using four different methods, namely Hilbert Transform (HT), Complex Demodulation (CD), Wavelet Transform (WT) and Matching Pursuit (MP). The six model parameters were subsequently estimated from the IE and IF waveforms. The average error, taking into account the error for each model parameter, was lowest for HT, higher but still less than 10% for CD and MP, and highest (greater than 10%) for WT, for three different spindle model examples. The amount of distortion induced by the use of a given method is also important; distortion was the greatest (0.4 sec) in the case of HT. Therefore, in the case of real spindles, one could utilize CD and MP and, if the spindle duration is more than 1 sec, HT as well.**

Keywords— **Sleep spindles, AM-FM signals, instantaneous envelope, instantaneous frequency**

I. INTRODUCTION

The sleep spindle waveform is one of the hallmarks of human stage 2 sleep EEG and is also one of the few transient EEG events which is unique to sleep. It is commonly known as a group of rhythmic waves within the frequency range of 11-15 Hz, characterized by a progressively increasing, then gradually decreasing, amplitude, which gives the waveform its characteristic name. It is usually of 0.5-2 sec duration, and it may be present in low-voltage background EEG, or superimposed to delta EEG activity, or temporally locked to a K complex [1]. The function of sleep spindles is currently under investigation, and several clinical applications of metrics derived from sleep spindle analysis have been reported in the literature [2]. This paper contributes to the quantification and modeling of the time-varying sleep spindle microstructure which may have clinical significance [3].

Sleep spindles can be regarded as AM-FM signals and can be expressed as [4]:

$$f(t) = A(t)\cos[g(t)]; A(t) \geq 0 \quad (1)$$

where: $A(t) = A_0 + k_a \cos(2\pi f_a t + \theta_a)$, as a simple approximation, is a model for the instantaneous envelope (IE) and $g(t) = 2\pi f_0 t + k_b \cos(2\pi f_b t + \theta_b)$, as a simple approximation, is a model for the instantaneous phase. The instantaneous frequency (IF), in rads/sec, is the time derivative of *g(t)*.

Previous work on the analysis and parameterization of sleep spindles has included the use of Complex Demodulation (CD) [4], [5] and Matching Pursuit (MP) techniques [6]. In addition to this, Hilbert Transform (HT) and Wavelet Transform (WT) methods have been used to analyze neuronal signals [7]. The above four methods are based on two distinct approaches which allow a direct estimation of the instantaneous envelope and instantaneous phase of a signal. Accordingly, the envelope and phase can be either estimated by using the analytic signal concept (HT and CD) or, alternatively, by convolution with a complex wavelet (WT and MP).

The aim of this paper was to compare the estimation of six parameters from the IE and IF waveforms, which can model the time-varying microstructure of sleep spindles based on the model of Eq. (1), in artificial signals simulating sleep spindles, using HT, CD, WT and MP. Based on the analysis of the simulated signals, practical recommendations are provided for the use of HT, CD, WT and MP in analyzing and modeling real sleep spindles for clinical applications.

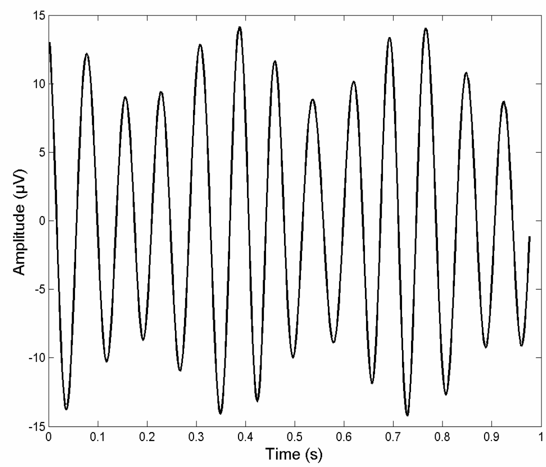
II. METHODOLOGY

A. Simulation of sleep spindles

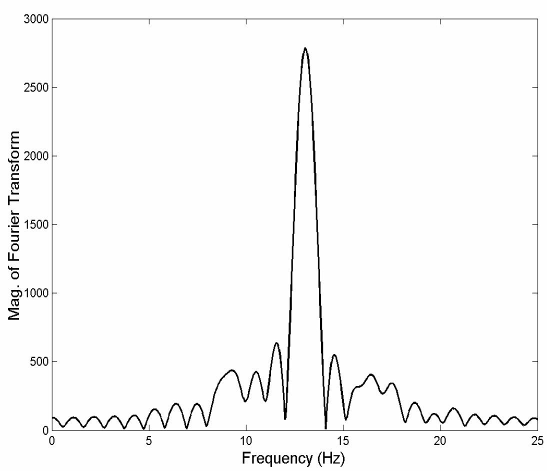
Given a sleep spindle signal, related IE and IF waveforms can be obtained from each of the four methods, and the six parameters, A_0, k_a, f_a, f_0, k_b, f_b, which model the time-varying microstructure of sleep spindles according to Eq. (1), can be estimated from the IE and IF waveforms by using the related models *A(t)* and *g(t)*. An example of a simulated sleep spindle signal is shown in Figure 1a, where the values of the six parameters used in the simulation are given below (the phase parameter values, θ_a, θ_b, are assumed to be zero):

f_0=13 Hz, A_0=11.5 microvolts, k_a=2.8 microvolts, f_a=2.7 Hz, k_b=0.27 rads, and f_b=4 Hz, where f_0 is the central frequency of the signal. These are typical values for a real spindle, based on preliminary work [5].

Figure 1a shows 500 points of the signal for a sampling frequency of 512 Hz. This signal length (about 1 sec) is typical for a sleep spindle. Figure 1b shows the one-sided Fourier Transform (FT) of the signal. In this graph, there is a peak at 13 Hz, as expected.



(a)



(b)

Fig. 1. Example of (a) AM-FM signal and (b) its one-sided Fourier Transform.

Furthermore, two additional signal examples were produced, with parameters within the range of typical values, as shown in Table 1.

B. Estimation of IE and IF of simulated sleep spindles

Hilbert Transform. HT is a well-known method to determine the instantaneous characteristics of a real signal. Accordingly, the IF function of a real signal $x(t)$ is defined as:

$$IF(t) = \frac{1}{2\pi}\frac{d\phi}{dt} \quad (2)$$

where ϕ is the phase of the analytic signal corresponding to $x(t)$. The analytic signal $Y(t)$ corresponding to $x(t)$ is defined by:

$$Y(t) = x(t) + i \cdot \hat{x}(t) \quad (3)$$

where $\hat{x}(t)$ is the HT of $x(t)$. The IE can be computed from the magnitude of the analytic signal:

$$IE(t) = \sqrt{x^2(t) + \hat{x}^2(t)} \quad (4)$$

Complex Demodulation. The procedure for applying CD to a given real signal $x(t)$ can be summarized as follows:

(1) The frequency spectrum of the signal is shifted to the left (towards the origin) by the demodulating frequency (assumed to be f_0 for the simulated signal).

(2) The resulting complex signal is filtered with a lowpass filter (for our case an FIR filter, order N=100, implemented using MATLAB's *FIR1*-command) having a cutoff frequency of f_c Hz. This cutoff frequency is chosen such that, beyond that frequency, the frequency content of $x(t)$ may be considered negligible.

(3) The frequency spectrum of the filter output is shifted to the right by the demodulating frequency (assumed to be f_0 for the simulated signal).

It is obvious that the objective of steps 1-3 above is to produce the analytic signal corresponding to the real signal $x(t)$. The instantaneous envelope and phase (as well as frequency) functions of $x(t)$ are subsequently calculated from the magnitude and argument, respectively, of the complex function that results after the last step. Thus, we obtain the IE and IF waveforms.

Wavelet Transform. For a given signal $x(t)$, the Continuous Wavelet Transform (CWT) $W(\alpha,\tau)$ of $x(t)$ is defined as the convolution between $x(t)$ and dilated versions of a complex wavelet function $\psi(t)$.

$$W(\alpha,\tau) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}\psi^*\left(\frac{t-\tau}{\alpha}\right)x(t)dt \quad (5)$$

where $\alpha \in \Re^+$ is the scale dilation parameter and $\tau \in \Re$ is the translation parameter. In the present study, the complex Morlet wavelet $\left(\psi_{Morlet}(t) = 1/\sqrt{\pi f_b}\, e^{j2\pi f_c t-(t^2/f_b)}\right)$ was used, where $f_c=1$ is the center frequency and $f_b=1$ is the bandwidth (variance). The scales examined were $23 \le \alpha \le 60$, with step equal to 1. It is possible to relate the scales α to the frequencies f by approximating the center frequency f_c of the wavelet using the relation $f_a = f_s f_c / \alpha$; where f_s is the sampling frequency.

For a monochromatic signal, there is a scale $\alpha_r(\tau)$ at any given τ which corresponds to a basis wavelet centered at τ whose frequency is equal to the local frequency of $x(t)$. The scale $\alpha_r(\tau)$ identifies the instantaneous frequency of the signal. As the CWT is a linear operation, superimposed frequency components are manifested as different scales $\alpha_r^{(i)}(\tau)$ which locally maximize $|W|$. The curves formed by the points $(\alpha_r^{(i)}(\tau),\tau)$ are known as the "ridges" of the transform. The trajectory of each ridge can be used to extract the IE and IF waveforms [8].

Matching Pursuit with moving window. In MP the aim is to decompose a given signal S into a linear combination of N waveforms (atoms) chosen from a predefined dictionary D_k so that:

$$S = \sum_{i=1}^{N} w_i d_i + R_N \qquad (6)$$

where R_N is the decomposition residual. In our case we used the following cosine-based dictionary:

$$D_k = \cos(2\pi f_k t + \phi_k) \quad (7)$$

where $f_k = 10:0.05:16$ and $\phi_k = 0:\frac{\pi}{10}:2\pi$

MP was performed within a moving 50-sample window, shifted every 5 samples (with N=1 in order to find the $d_i = \cos(2\pi f_i t + \phi_i)$ that best matched the signal).

The parameters obtained from the application of MP in every window (frequencies f_k and weights w_i) were then interpolated and smoothed with a cubic smoothing spline in order to produce the instantaneous frequency (IF) and instantaneous envelope (IE) curves, respectively.

C. Estimation of model parameters

The six model parameters mentioned in Section IIA were obtained from the estimated IE and IF waveforms. To this end, only a part of the waveform was used, resulting after subtracting distortions induced by the use of each method (e.g., FIR filter distortion in CD).

To evaluate the performance of the above four methods, the errors corresponding to the estimation of the six model parameters were calculated. In addition, an average error per method was also calculated.

III. Results

Table1 1 shows the six model parameters estimated using HT, CD, WT and MP on the three simulated AM-FM signals. As we can see, HT had the lowest errors in the estimation of the parameters. The average errors of CD and MP were lower than 10%. The WT method resulted in the highest errors; on the average, these errors were higher than 10%. Across all methods, the highest errors were observed for the estimation of k_b, which relates to the amplitude of the IF waveform; they reached 65% and 50% for WT and MP, respectively, in spindle example 3. The lowest errors were observed for f_a, which is the frequency of the IE waveform.

The amount of distortion that was subtracted from the IE and IF waveforms before further processing (estimation of model parameters) was 0.40 sec for HT and 0.26 sec for CD.

TABLE 1. ESTIMATED MODEL PARAMETERS USING HT, CD, WT AND MP AND ERRORS COMPARED TO THEORETICAL VALUES.

	Theory	**HT**	**Error – HT**	**CD**	**Error - CD**	**WT**	**Error - WT**	**MP**	**Error – MP**
Spindle example 1									
f_0	13.00	13.00	0.00%	12.69	2.35%	14.17	9.00%	13.06	0.40%
A_0	11.50	11.52	0.10%	11.41	0.78%	11.95	3.91%	11.67	1.48%
f_a	2.70	2.81	3.70%	2.69	0.2%	2.65	3.33%	2.69	0.03%
f_b	4.00	4.06	1.50%	4.02	0.39%	3.83	4.25%	4.19	4.75%
k_a	2.80	2.78	0.70%	2.46	12.14%	2.51	10.36%	2.65	5.36%
k_b	0.27	0.27	0.00%	0.18	33.33%	0.16	40.74%	0.24	11.11%
Average error			1.00%		8.20%		11.93%		3.85%
Spindle example 2									
f_0	12.00	12.02	0.17%	11.71	2.42%	12.96	5.75%	12.27	2.25%
A_0	8.00	8.04	0.50%	7.87	1.63%	7.85	1.88%	8.11	1.37%
f_a	2.50	2.61	4.40%	2.47	1.20%	2.46	1.60%	2.50	0.00%
f_b	3.50	3.31	5.43%	3.47	0.86%	3.71	6.00%	3.59	2.57%
k_a	2.00	2.00	0.00%	1.85	7.50%	0.88	56.00%	1.92	4.00%
k_b	0.40	0.42	5.00%	0.34	15.00%	0.26	35.00%	0.29	27.50%
Average error			2.60%		4.77%		17.70%		6.26%
Spindle example 3									
f_0	13.50	1.3.50	0.00%	13.21	1.62%	13.62	4.77%	13.71	1.56%
A_0	12.00	12.01	0.08%	11.95	0.42%	12.31	2.58%	12.18	1.50%
f_a	3.00	3.01	0.33%	3.01	0.33%	2.85	5.00%	2.99	0.33%
f_b	4.50	4.53	0.67%	4.49	0.22%	4.10	8.89%	4.53	0.67%
k_a	3.00	3.00	0.00%	2.64	12.00%	1.47	51.00%	2.77	7.67%
k_b	0.20	0.20	0.00%	0.15	30.00%	0.07	65.00%	0.10	50.00%
Average error			0.17%		7.43%		22.87%		10.29%

IV. Discussion

In this paper, four time-frequency analysis methods, namely HT, CD, WT, and MP, were used to model the time-varying microstructure of AM-FM signals simulating sleep spindles. The estimated model parameters are believed to be of clinical significance and, therefore, are expected to change in the presence of pathology (eg. dementia, depression), as inferred by previous studies [2]. Preliminary work has found that these parameters provide an efficient global quantification of the visual impression regarding overall spindle morphology during human sleep staging [5]. In addition, earlier work indicates that some of these parameters should be affected by age [3].

The relative comparison of the four methods should be based not only on the accuracy of the model parameter estimation but also on criteria such as the distortion in the signal induced by a given method. This is particularly important in the case of real sleep spindles, which are of limited duration and rarely reach a length more than 1.5 sec [9].

In terms of accuracy, the application of the four methods in three different signal models showed that HT had the lowest errors in the estimation of the model parameters. CD and MP had errors lower than 10%. Thus, these three methods were shown to quantify in a relatively accurate way the microstructure of simulated sleep spindles. WT showed the highest errors (on the average, higher than 10%). However, since the above data are preliminary, more detailed analysis of the methods with more examples is needed.

The amount of distortion induced in the obtained IE and IF waveforms was highest in the case of HT, where 0.4 sec were subtracted from the IE and IF waveforms before further analysis. On the other hand, for CD only 0.26 sec were subtracted. This is an important constraint for the use of HT in the analysis of real spindles of short duration, e.g. 0.6-1 sec.

A number of limitations should be mentioned related to the implementation of the four methods. CD is basically a heuristic method, related to analytic signal estimation, and its accuracy depends on the optimal selection of the demodulating frequency and of the lowpass filter cutoff frequency (f_c). Ideally, the choice of f_c is a trade-off between two actions: first, to allow as much signal energy as possible to remain after filtering and second, to prevent unwanted signal content (noise) to remain for further analysis. Obviously, the optimal choice of the demodulating and of the cutoff frequency is crucial for the analysis of real signals, and that choice should be based on the heuristic inspection of the FFT of a real spindle signal. On the other hand, the HT methodology, which also relates to analytic signal estimation, is fairly straightforward.

For MP, the choice of an atom dictionary, as well as the related resolution, should affect the results. For WT, there are intrinsic edge artifacts [10], because the Morlet wavelet is not completely localized in time, which affect the parameter estimation. In addition, a more smooth choice of scales around an expected f_0 could improve the results. Different mother wavelets could also be investigated for error reduction.

V. Conclusion

The time-varying microstructure of sleep spindles can be quantified in terms of a set of six parameters, according to a simple spindle model, in a relatively accurate way by using HT, CD and MP. Apparently, HT is associated with the lowest error in the estimation of the model parameters, but induces the greatest amount of distortion in the resulting IE and IF waveforms. Therefore, in the case of real spindles, one could utilize CD or MP and, if the spindle duration is more than 1 sec, HT as well.

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