Regularization Method

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Abstract- The temperature recovering curve derived by cold stimulation experiment reveals human inner metabolism status. The errors of the measured date should be eliminated. A Tikhonove regularization method is used in this study to regularize the original data. The results present that the ability of body metabolic function can be measured by the temperature derivative curve, especially the characteristics in the first stage. Two typical shapes of these derived curves reflect the difference of two kinds of blood vessels in clinic. These temperature derivative curves oscillate generally in the last stage.

Keywords- regularization, metabolism, temperature derivative

I INTRODUCTION

The original collected data, including clinic data and experiment data, are very significant in biomedical signal processing. Inaccurate original data will result in serious mistaken analysis. The inaccuracy always results from the measure error of machine and man-made influence. Once the error exceeds an acceptable limit, the measured data will be unusable. So, data-filtering is the prerequisite for further data process.

During the past six years, a novel metabolic function parameter is proposed based on cold stimulation experiment[1-3]. This parameter could evaluate the inner metabolism status quantitatively. By systematic research, it is found that the measured body surface temperature is contaminated by errors. When the temperature derivative is considered, the original data present a big disorder, as showed in Fig.1. In Fig.1, blue point is the original temperature value and the red weighted curve is its corresponding derivative. Here, an infrared thermograph (TH5108ME, made by NEC san-ei Instrument, ltd, Japan) is used to record temperature in the experiment. In practice, the longer the distance between hands surface and the receive screen is, the larger the difference between two neighboring points of the infrared image would be. The average distance error will reach ±0.2°C. The machine accuracy of thermograph is ± 0.1 °C. So, the total measure error of temperature is ± 0.3 °C. The temperature of human body is continuous and smooth. In order to improve the accuracy of the following research, it is necessary to eliminate the measure error as much as possible. In this paper the errors are filtered out by regularization method. This method(strategy) was advanced by Tikhonov and Philips in 1960s respectively[4]. They provided a useful

method to deal with discrete ill-posed problems and it has wide applications in many science engineering fields.

The outline of this paper is the following: the basic theory of regularization method is introduced in part two. And in part three, the regularized results are presented and the characteristics of metabolic activity curves are discussed. Finally, some concluding remarks are provided.

II METHOD

Given a linear discrete ill-posed problem [4-7]:

$$Ax = b , \qquad A \in \mathbb{R}^{m \times n} , \quad m \ge n \tag{1}$$

where A is an ill-conditioned matrix and b is a vector which is measured discretely with data errors.

The primary difficulty of the solution is highly sensitive to high-frequency perturbations. Hence, it is necessary to incorporate further information about the desired solution in order to stabilize the problem and single out a useful and stable solution. This is the purpose of regularization.

Undoubtedly, the most common and well-known form of regularization is the one known as Tikhonov regularization. It amounts to solving the problem:

$$\min_{x} \{ \| Ax - b \|_{2}^{2} + \lambda \| Lx \|_{2}^{2} \}$$
(2)

where λ is the regularization parameter, and the regularization operator L is a matrix that defines a suitable smoothing norm on the solution.

Consider the singular value decomposition (SVD) of the coefficient matrix A:

$$A = U \sum V^{T} = \sum_{i=1}^{r} \sigma_{i} u_{i} v_{i}^{T}, \quad r = rank(A)$$
(3)

then Tikhonov regularization in the form (2) provides a solution:

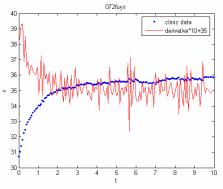


Fig. 1 Original data and its derivative.

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$$x_{reg} = \sum_{i=1}^{r} f_i \frac{u_i^T b}{\sigma_i} v_i \qquad \text{if} \quad L = I_n \quad (4)$$

$$x_{reg} = \sum_{i=1}^{p} f_i \frac{u_i^T b}{\sigma_i} x_i + \sum_{i=p+1}^{r} (u_i^T b) x_i \text{ if } L \in \mathbb{R}^{p \times n} \neq I_n$$
(5)

here, the numbers $f_i = \frac{\sigma_i^2}{\sigma_i^2 + \lambda^2}$ are filter factors for the particular regularization method. The filter factors must have the important property that as σ_i decreases, the corresponding f_i tends to zero in such a way that the contribution $\frac{u_i^T b}{\sigma_i} x_i$ to the solution from the smaller σ_i are effectively filtered out.

In the above regularization solution, noise in vector b is filtered out by the filter factors determined by a regularization parameter λ . So it is important to select an appropriate λ to obtain a reasonable solution. Several methods for choosing λ are popular in literature: the discrepancy principle, generalized cross-validation, L-curve, and so on.

Perhaps the most convenient graphical tool is L-curve. It is a plot of the (semi)norm $||Lx_{reg}||_2$ of the regularized solution versus the corresponding residual norm $||Ax_{reg} - b||_2$ for all valid regularization parameters. In this way, the L-curve clearly displays the compromise between minimization of these two quantities, which is the core of any regularization method. The L-curve is defined as:

$$(\xi(\lambda), \eta(\lambda)) \equiv (\log(||Ax - b||), \log ||Lx||)$$
(6)

The graph of the curve looks like the letter "L". The value of the parameter λ corresponds to the point at the "vertex" of the "L", where the vertex is defined to be the point on the L-curve with curvature c_{λ} of largest magnitude. *c* is defined as:

$$c(k) = \frac{\xi' \eta'' - \xi'' \eta'}{\{(\xi')^2 + (\eta')^2\}^{3/2}}$$
(7)

In this paper, b is the measured body surface temperature, x is the accurate temperature we want to obtain, A could be predefined to be a unit matrix. The purpose is to get an optimal smooth solution which could maintain the information about human body as much as possible. This is a typical regularization problem. Here Lis a predefined banded matrix with full row rank:

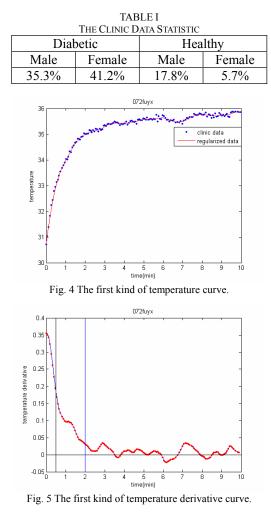
$$L(p,n) = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix}$$

III RESULTS

Cooperating with Tongji Hospital we have collected 1884 cases clinic data about the diabetics and the healthy from October 10, 2001 to January 11, 2006. The distribution is showed in Table I. In this paper, the characteristics of their recovering curves are analyzed.

In the research process, we found the recovering curve has two different kinds. The first kind represents those whose hands feel hot in winter and the temperature curve is showed in Fig.4. Fig.5 is its corresponding temperature derivative curve processed by regularization method. Likewise, the second represents those whose hands feel cold in winter and the temperature curve is showed in Fig.6. Fig.7 is its corresponding temperature derivative curves processed by regularization method. With a large amount of statistic data analysis, it is found these two different figures have no relation with age, gender, and whether or not diabetic.

In Fig.4 and Fig.6, the blue points denote measured temperature and the red line is the regularized curve. In Fig.5 and Fig.7, the red points denote the derivative of regularized data and the blue line is the derivative curve. Here time starts at the moment when hands are put out from the cold water. Form Fig.4 and Fig.6, the regularized curves follow the original data tendency very well. Comparing the



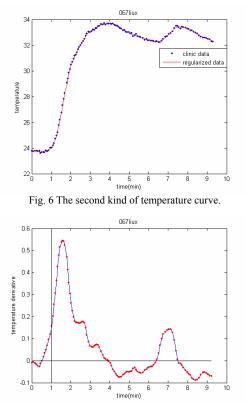


Fig. 7 The second kind of temperature derivative curve.

derivative curve in Fig.1 and Fig.5, the regularized curve is smoother. Much information about human metabolic activity can be obtained from these figures.

Firstly, by cold water stimulation, the first kind of temperature curve ascends to a normality immediately, which is showed in Fig.4. Conformably, in Fig.5, in the first thirty seconds the values of the derivative are the largest. It means in this period the temperature increases rapidly. This time lasts from about thirty seconds to ninety seconds according to individual subject characteristics respectively. However, the second kind of temperature curve is different. The temperatures will keep on a low value from the beginning to about sixty seconds before its ascending, which is showed in Fig.6. The derivative values of this kind in the first sixty seconds in Fig.7 are very small or even lower than zero, and the temperature increases very little or even decreases. This period lasts from about twenty seconds to one hundred and twenty seconds according to individual subject characteristics respectively as well. A reasonable explanation about this characteristic is the difference of blood vessel. The arteries of the first kind person are thick and the blood volume is high as well. In winter, the arteries maintain the blood expedite, which makes hands feel hot. After removing the cold water, the contractive arteries could expand as quickly as possible, and the body temperature ascends rapidly, so the recovering curve displays just like Fig.4. On the contrary, the arteries of the second kind person are thin and the blood volume is low. So, in winter, these arteries are contractive by cold air and the blood flow is blocked, which makes hands feel cold. After the stimulation removed, the arteries can not expand immediately, it needs more time to recover, and the recovering curve displays like Fig.6.

Secondly, the derivative curve depicts the temperature change ratios. It provides a quantitive index to estimate the metabolic function. The data in Fig.5 belong to a diabetic and the largest derivative value is about 0.35. The data in Fig.7 belong to a healthy and the largest derivative is above 0.5. It is known to all that the healthy metabolic ability is better than the diabetic. And the derivative value corresponds with this fact. The temperature change ratios derived from the heat transfer quantity of blood perfusion and heat-production quantity of metabolism. The derivative value is very useful to separate these two main components in the further research.

Thirdly, the derivative curve is wavy in the last period. It indicates the oscillation in the recovering process. Commonly, this oscillation is faint and disorder, which is showed from the second to the tenth minute in Fig.5. However, some oscillation is intense and rhythmic, as another clinic case showed clearly in Fig.8. In Fig.8, the blue points denote regularized data and the red points denote its weighted derivative. In the period from the fourth to the thirteenth minute, the shape of this derivative curve is very regular and looks like SIN signal. This characteristic is related with the rhythmic flexibility of the blood vessel. When the arteries are expanded, the blood volume increase and the temperature ascend and the derivative value is large as well. On the contrary, when the arteries are contractive, the blood volume decrease and the temperature descend and the derivative value is small under zero.

IV CONCLUSION

In this paper, regularization method is adopted to eliminate the error in infrared image data. And the changes in the temperature recovering curve are displayed more clearly by the method. The study results show three important points as follows:

1. The temperature recovering curve by cold stimulation can be classified into two types. Their derivative curves also present two different kinds. The characteristics depend on the value of the derivative curve in the first thirty seconds to ninety seconds. So the first stage character of derivative curve will be most helpful in clinic test.

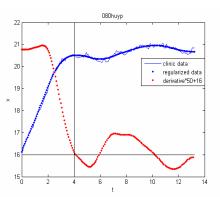


Fig. 8 Another clinic, regularized and temperature derivative data.

2. The derivative curve change ratios indicate the characteristics of human metabolism. The larger of this change ratio is, the better the metabolic function is. This characteristic will be helpful in further evaluating the serious of diabetics.

3. The regular oscillation of the regularized derivative is one of the typical characteristic. About 40% results comply with this characteristic which indicates that the recovering process in the last stage is stable but oscillatory.

The derived characteristics here are the foundations for separating different heat factors in further research. In this process, theoretical deduction and experiment analysis should have a proper combination.

REFERENCE

- Z.G. Shanh, G.T. Jiang, A Method of Hurtless Determination Tissue Metabolic Rate at Steady Temperature Field, 23th IEEE/EMBC, Oct.25-28, 2001.
- [2] Guotai Jiang, Zhigang Shang. A New Evaluation Technique of Metabolic Efficiency and the Application for Prevention and Cure of Diabetes, World Engineers' Convention 2004, Nov. 2-6, 2004.
- [3] Hai Liu, Guotai Jiang, Zhigang Shang, The separation of metabolizing heat based on independent component analysis, 27th IEEE/EMBC, Sep.1-4, 2005.
- [4] A.N. Tikhonov, Solution of incorrectly formulated problems and the regularization method, Soviet Math. Dokl., 4(1963), pp.1035-1038.
- [5] P.C. Hansen, Analysis of discrete ill-posed problems by means of the L-curve, SIAM Rev., 34(1992), pp. 561-580.
- [6] D. Calvetti, L. Reichel, A. Shuibi, L-curve and curvature bounds for Tikhonov regularization, Numerical Algorithms, 35(2004), pp.301-314.
- [7] P.C. Hansen, Regularization Tools: A Matlab package for analysis and solution of discrete ill-posed problems, Numer. Algo., 6(1994)pp. 1-35.