Proceedings of the 28th IEEE
EMBS Annual International Conference
New York City, USA, Aug 30-Sept 3, 2006

Capacitive Electrodes in Electroencepahlography

Nicolás von Ellenrieder nellen@ieee.org

Enrique Spinelli spinelli@ing.unlp.edu.ar

Carlos H. Muravchik carlosm@ing.unlp.edu.ar

Laboratorio de Electrónica Industrial, Control e Instrumentación, Área Departamental Electrotecnia, Facultad de Ingeniería, Universidad Nacional de La Plata, C.C.91, 1900 La Plata, Argentina.

***Abstract*— We present a forward problem formulation for computing biopotentials measured with dry or capacitive electrodes. This formulation is not quasistatic and has mixed boundary conditions. Our results show that simple approximations to the measurements based on capacitive coupling are adequate in most situations. We study the range of validity and errors committed in the EEG forward and inverse problems when using this approximation.**

I. INTRODUCTION

The use of dry or capacitive electrodes in the measurement of biopotentials was proposed several decades ago [1] and continues to be of interest [2]. Dry electrodes avoid the electrochemical skin-gel-metal interface by placing an insulating material between the skin and the electrode. The potential is measured then through capacitive coupling, i.e. the potential of the electrode is assumed to be equal to the potential on the skin, under the electrode. This assumption has been tested in many practical cases, but to the authors knowledge no theoretical formulation of the problem has been made. We present a formulation that is not quasistatic and has mixed boundary conditions, and use it to determine the range of validity of the capacitive coupling assumption in the EEG forward and inverse problems.

II. PROBLEM FORMULATION

The quasistatic approximation of Maxwell's equations holds for the EEG forward problem with conventional electrodes because the head size is much smaller than the wave length, and the displacement current is much smaller than the ohmic current in all the tissues of the head. For dry electrodes the first condition holds (i.e. the electric and magnetic fields remain uncoupled), but the quasistatic approximation is not valid because displacement currents are present in the insulating layer between the scalp and the electrode.

Under these conditions the differential problem is time dependent, and adopting a layered head model, it can be formulated in the Laplace domain as

$$\varsigma \nabla^2 \phi(\boldsymbol{x}) = \boldsymbol{\nabla} \cdot \boldsymbol{J}_p(\boldsymbol{x}), \tag{1}$$

where $\phi(\boldsymbol{x})$ is the electric potential on point $\boldsymbol{x}$, $\boldsymbol{J}_p(\boldsymbol{x})$ is the primary current forced by the source in point $\boldsymbol{x}$, and $\varsigma = \sigma + s\epsilon$ is a "complex conductivity" (σ is the electric conductivity, ϵ the electric permittivity and s the Laplace operator).

N. von Ellenrieder and E. Spinelli are with CONICET. C. Muravchik is with CICPBA. This work was supported by ANPCyT PICT 11-14111.

The boundary conditions are given by

$$\begin{cases} \phi(\boldsymbol{x}^+) = \phi(\boldsymbol{x}^-) & \boldsymbol{x} \in S_i \\ \varsigma^+ \boldsymbol{\nabla}\phi(\boldsymbol{x}^+)\cdot\boldsymbol{n} = \varsigma^- \boldsymbol{\nabla}\phi(\boldsymbol{x}^-)\cdot\boldsymbol{n} & \boldsymbol{x} \in S_i \\ \phi(\boldsymbol{x}) = \Phi_k & \boldsymbol{x} \in S_{ek} \end{cases} \tag{2}$$

where $\boldsymbol{x}^-$ and $\boldsymbol{x}^+$ are the limit points just inside and outside the interface S_i, $i = 1, \dots M$, $\boldsymbol{n}$ is the outward point normal on the interfaces, S_{ek}, $k = 1, \dots K$ are the interfaces between the insulating material and the electrodes, and Φ_k is the electric potential on electrode k (i.e. the unknowns to be determined). The constraints

$$\int_{S_{ek}} \boldsymbol{\nabla}\phi(\boldsymbol{x})\cdot\boldsymbol{n}\, da = 0 \qquad \boldsymbol{x} \in S_{ek} \tag{3}$$

must also be observed to model an infinite input impedance of the amplifiers [3]. Note that for the head tissues $\varsigma \approx \sigma$, and for the insulating material and air $\varsigma \approx s\epsilon$. In fact, if the permittivity of the insulating material is several times larger than the permittivity of vacuum, the "complex conductivity" (ς) of the air can be neglected. Also, in this case it is not necessary to model the electrode, only the interface between the insulator and the metal. Hence, the model only needs to include the layers representing the head tissues and the insulating patches.

It is possible to find an integral formulation of the problem, through Green's Second Identity. The resulting equation is similar to the classic EEG forward problem integral formulation, but with the electric conductivity σ replaced by ς, and extra terms to account for the Neuman boundary conditions of the electrodes. Since the value of ς for the insulating material is much smaller than for the head tissues, it is necessary to use the Isolated Problem Approach [4] to obtain a correct solution of the forward problem. The integral problem solution is then given by $\phi(\boldsymbol{x}) = \phi_0(\boldsymbol{x}) + \varphi(\boldsymbol{x})$, where $\phi_0(\boldsymbol{x})$ is the electric potential in the head when no electrodes are present and zero outside the head (i.e. standard forward problem solution), and $\varphi(\boldsymbol{x})$ is given by

$$\begin{aligned} c_i(\boldsymbol{x})\varphi(\boldsymbol{x}) = &\sum_{m=1}^{M} \frac{(\varsigma_m^+ - \varsigma_m^-)}{2\pi} \int_{S_m} \varphi(\boldsymbol{y}) \boldsymbol{\nabla} \frac{1}{|\boldsymbol{x}-\boldsymbol{y}|} \cdot \boldsymbol{n} da(\boldsymbol{y}) \\ &+ \sum_{k=1}^{K} \frac{\varsigma_M^-}{2\pi} \int_{S_{ek}} \left(\Phi_k \boldsymbol{\nabla} \frac{1}{|\boldsymbol{x}-\boldsymbol{y}|} \cdot \boldsymbol{n} - \frac{\boldsymbol{\nabla}\varphi(\boldsymbol{y}) \cdot \boldsymbol{n}}{|\boldsymbol{x}-\boldsymbol{y}|} \right) da(\boldsymbol{y}) \\ &+ \varsigma_M^- \left(\int_{S_{M-1}} \frac{\phi_0(\boldsymbol{y})}{2\pi} \boldsymbol{\nabla} \frac{1}{|\boldsymbol{x}-\boldsymbol{y}|} \cdot \boldsymbol{n} da(\boldsymbol{y}) - \delta_{i,M-1}\phi_0(\boldsymbol{x}) \right) \end{aligned} \tag{4}$$

where $\boldsymbol{x} \in S_k$, $c_i(\boldsymbol{x})$ is the geometric mean of ς at point $\boldsymbol{x}$, $\delta_{i,M-1}$ is a Kronecker delta, S_{M-1} is the set of scalp-insulator interfaces and ς_M^- corresponds to the dielectric material. The last term, which is a function of ϕ_0, acts as the source term.

We solve the integral solution by tesselating the surfaces in a set of triangles and using COG BEM [4]. The solution of this problem yields the electric potential on the surfaces S_i and the normal component of its gradient on surfaces S_{ek}. The potential on the electrodes are extra variables, and the constraints (3) are needed to complete the linear system. If the reference is not chosen *a priori* the system is singular and should be solved using a deflation technique. Another option is to choose one of the electrodes as the reference.

III. RESULTS

We adopted a dipolar source model and three layered spherical head model, with radii of 87, 92 and $100mm$, and electric conductivities 0.33, 0.004125 and $0.33S/m$ for the brain, skull and scalp. The electric potential on the surface of the head when no electrodes are present was computed analytically [5], and used as the source term in (4). Unless stated otherwise, the insulating patch of the dry electrodes was $1mm$ thick, with $\epsilon{=}10\epsilon_0$.

First we solved the problem for different frequencies (i.e. with $s{=}j2\pi f$) and found that the frequency response of the system is flat, i.e. has negligible delay, well beyond the frequencies of interest in EEG. For instance, even with a permittivity $\epsilon = 10^4\epsilon_0$, the phase delay of the solution was negligible up to $100KHz$. The forward problem may then be solved as time independent, as when adopting the quasistatic approximation.

We also found that the presence of the dry electrode does not change the electric potential distribution in the head. The variation of the electric potential computed with (4) is more than six orders of magnitud smaller than the actual potential distribution.

The results mentioned in the previous paragraph support the idea that a good approximation to the electrodes potential can be obtained from the electric potential on the scalp, not taking into account the effects of the dry electrode. The usual approximation is to consider that the potential measured by the electrodes is $\Phi_k = \phi(\boldsymbol{x}_k)$, where $\boldsymbol{x}_k$ is the point on the scalp surface right under the center of the electrode. We will refer to this approximation as Approximation A.

Since the electric potential is fixed for the electrode but may vary under it, we propose another approximation for the measured potential, $\Phi_k = \int_{S_{uk}} \phi(\boldsymbol{x}) da/A$, where A is the area of the metallic electrode, and S_{uk} is the scalp surface under electrode k. This Approximation B is given then by the mean value of the electric potential under the electrode.

In the following sections we study the performance of the proposed approximations.

A. Single Electrode

In this section we study the error of Approximations A and B in the determination of the electric potential measured by a single electrode. We adopted an hexagonal electrode placed on the top of the head model, with an area of $33mm^2$, i.e. $10mm$ radius.

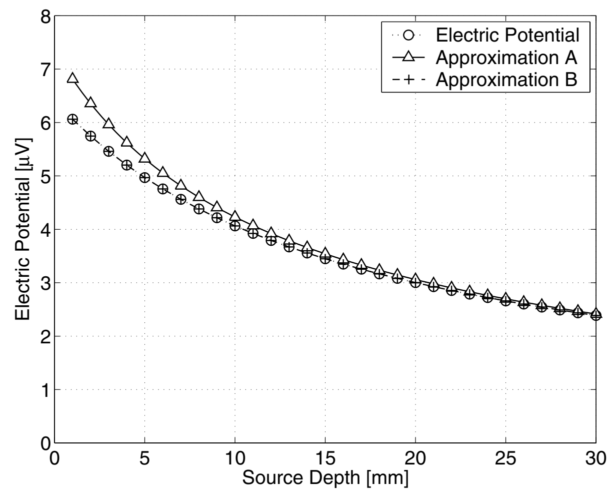

Fig. 1. Electric potential on a capacitive electrode due to a radially oriented dipolar source under the electrode versus source depth. Approximation A is the potential under the center of the electrode and Approximation B the mean value of the potential under the electrode.

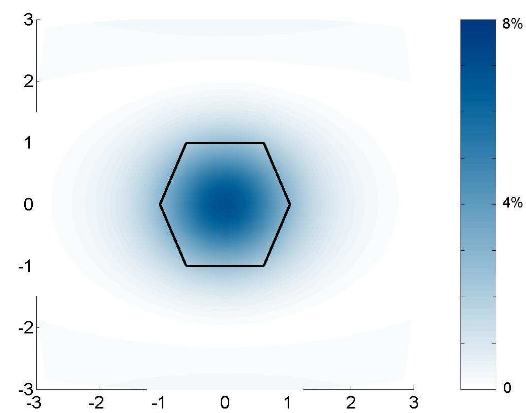

Fig. 2. Relative error of Approximation A as a function of source position. Radial source at $5mm$ depth. The black lines are the outline of the electrode.

In Fig. 1 we show the potential of the electrode as a function of the source depth under the surface of the brain. The source is a radially oriented dipole of $20nAm$ intensity located under the center of the electrode. The figure shows the true value of the potential computed with (4), and both approximations. As expected, Approximation A has a larger error for sources near the surface of the brain, and Approximation B produces better results.

In Fig. 2 we show the relative error of Approximation A as a function of the source location. The source is a radially oriented dipole located at a constant depth of $5mm$ under the surface of the brain. The error has a maximum value of 8% under the electrode. The black lines show the outline of the sensor.

The error of Approximation B is lower, it remains un-

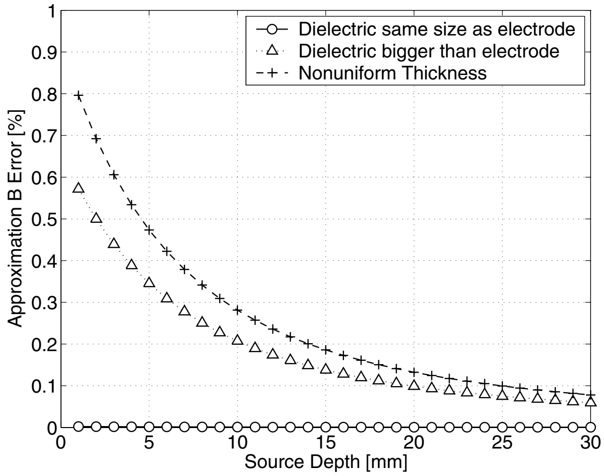

Fig. 3. Relative error of Approximation B as a function of source depth. Radial source under the center of the electrode.

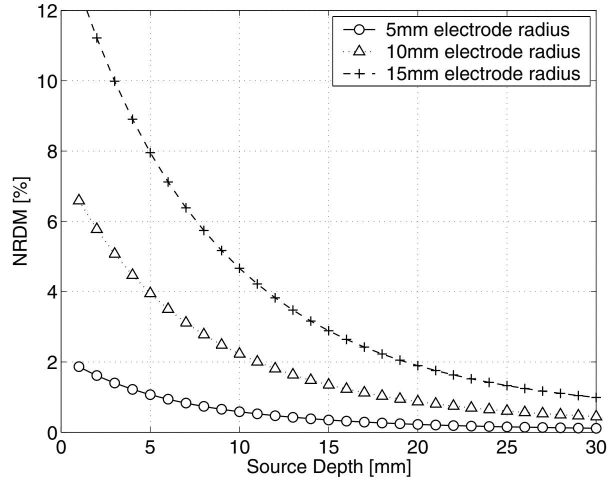

Fig. 4. NRDM of forward problem solution as a function of source depth for a radial source under electrode Cz. Results for different electrode radii.

der 1% in many different situations. Fig. 3 shows the relative error of Approximation B for a radially oriented dipolar source under the center of the electrode. The error increases when the insulating patch is larger than the electrode since the fringe effects increase. The approximation is not very sensitive to changes in the thickness of the dielectric patch, the error is lower than 1% for a nonuniform thickness of $1mm$ in the center and $1.75mm$ in the rim.

The results for tangentially oriented dipoles was similar, with a little larger relative errors but lower absolute errors. The sources located under the electrodes rim show the maximum value of the absolute error.

While approximation B is very good in all tested situations, Approximation A shows errors of up to 8% for a single electrode. Nevertheless, since the error is large near the electrode, the overall error in the forward problem when more sensors are used may not be too important.

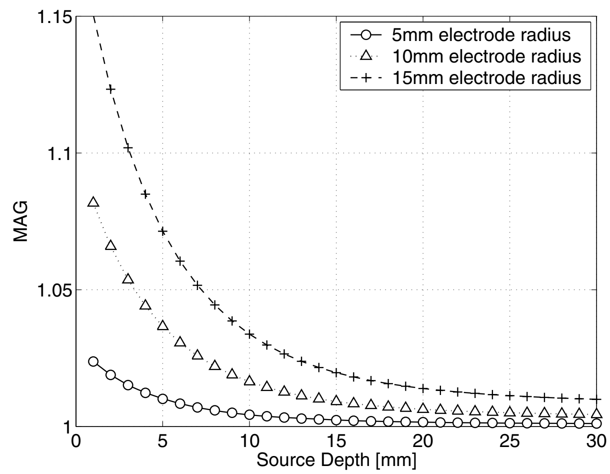

Fig. 5. MAG of forward problem solution as a function of source depth for a radial source under electrode Cz. Results for different electrode radii.

B. Forward Problem

To study the error of Approximation A when several sensors are used, we solved the forward problem for a set of 63 sensors located on the surface of the scalp according to the 10-20 system. Approximation B was used as the reference given its excellent performance shown in the previous section. The mean value of the electric potential under the electrodes was computed using 2D Gaussian quadrature, with the value of the potential on the quadrature points computed analytically [5].

We use the NRDM and MAG error measures to quantify the error of Approximation A on the forward problem. Their are given by

$$NRDM = \| (\boldsymbol{\Phi}_A/\|\boldsymbol{\Phi}_A\|) - (\boldsymbol{\Phi}_B/\|\boldsymbol{\Phi}_B\|) \| \quad (5)$$

$$MAG = \|\boldsymbol{\Phi}_A\| / \|\boldsymbol{\Phi}_B\|, \quad (6)$$

where $\boldsymbol{\Phi}_A$ and $\boldsymbol{\Phi}_B$ are vectors formed by the potential on the electrodes computed with Approximations A and B.

Fig. 4 shows the NRDM and Fig. 5 the MAG for a radially oriented source under the center of one of the electrodes (Cz), as a function of source depth. The electrodes are circular and the curves show the error for different radii of the electrodes. As expected, the error increases for larger electrodes, since the Approximation A assumes a point-like electrode.

C. Inverse Problem

In this section we study the effect of solving the inverse problems with the proposed approximations. First we study the systematic error committed with Approximation A by computing the misslocalization error, i.e. the distance between the real and estimated position of the dipolar source in a noise free situation. This was done by computing the forward problem solution $\boldsymbol{\Phi}_B$ with Approximation B, and then finding the source parameters (position, orientation and intensity) that produces a forward problem solution $\boldsymbol{\Phi}_A$ with Approximation A which minimizes $\|\boldsymbol{\Phi}_A - \boldsymbol{\Phi}_B\|$. A standard successive approximations algorithm was used.

In the presence of noise the true and the estimated source

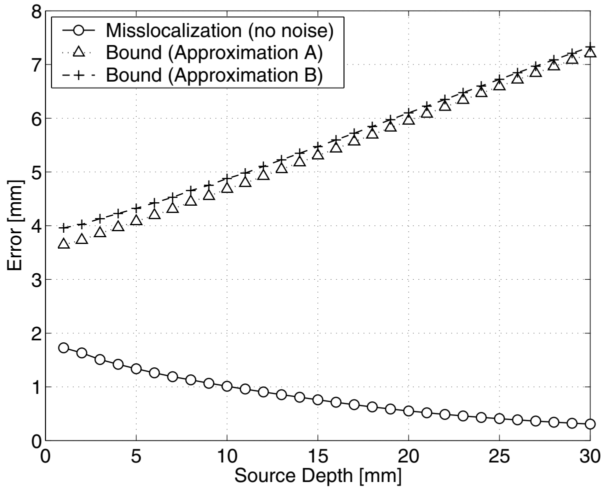

Fig. 6. Misslocalization systematic error and equivalent radius of the minimum 90% probability ellipsoid computed with Approximations A and B, as a function of source depth. Radial source under electrode Cz.

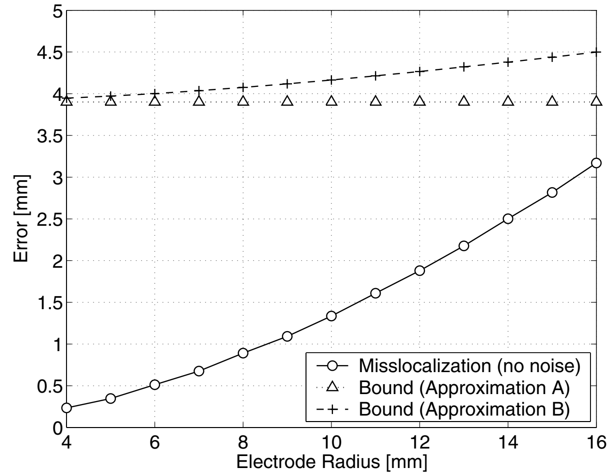

Fig. 7. Misslocalization systematic error and equivalent radius of the minimum 90% probability ellipsoid computed with Approximations A and B, as a function of electrode radius. Radial source at $5mm$ depth under electrode Cz.

positions are in general different. If the noise distribution is known, it is possible to find an ellipsoid centered in the estimated source position, and containing the source with 90% probability. The volume of this ellipsoid depends on the estimation error variance. We computed the Cramér-Rao Bound, which gives a lower limit for the variance of any unbiased estimator of the source parameters, and obtained a lower limit for the volume of the 90% probability ellipsoid [6]. Then we compute the equivalent radius of this ellipsoid (the radius of a sphere with the same volume). In this way we determine if the systematic error is significant in the presence of noise. We adopted a Normally distributed noise model, independent among sensors, with zero mean and $0.4\mu V$ standard deviation, which is a typical model of the electronic noise of the amplifiers.

Fig. 6 shows the misslocalization systematic error for a radially oriented dipolar source under the center of one of the electrodes (Cz) as a function of the depth of the source. The electrodes configuration is the same than in the previous section, and their radius is $10mm$. We also show the equivalent radius of the minimum 90% probability ellipsoid computed with Approximations A and B. We see that the systematic error is not very significant in this situation. Since the position and orientation of the source correspond to a worst case scenario, we can conclude that for electrodes of $10mm$ radius the Approximation A can be used safely to solve the EEG inverse problem with capacitive electrodes.

Fig. 7 shows the same variables a as function of the electrodes radius, for a fixed source depth of $5mm$ under the surface of the brain. We see that for larger electrodes the systematic error is comparable to the noise effect and thus Approximation B should be used to solve the inverse EEG problem. The figure also shows a mild increment of the estimator variance when the area of the electrodes increases, which indicates a loss of resolution of larger electrodes.

IV. CONCLUSION

In this work we showed that the usual approximation adopted for dry electrodes measurements as point-like electrodes on the scalp produces corrects results of the EEG inverse problem for electrodes with radius up to $10mm$. For bigger electrodes the mean value of the electric potential under the electrode should be used.

It is important to note that while the presented formulation shows that for a given area the electric potential on the electrode is independent of the electrode capacity, from the amplifier point of view a high electrode capacity is desirable because of the presence of stray capacities. Thus the bigger the electrode the more precise the measurement.

Since the capacitive electrodes do not change the electric potential distribution on the scalp, it would be wise to cover the entire surface with electrodes. We plan to follow the ideas presented in this work to find a trade-off between a large number of small electrodes or a lower number of larger, more accurate, electrodes when solving the inverse problem.

REFERENCES

[1] W. H. Ko, M. R. Neuman, R. N. Wolfson, and E. T. Yon, "Insulated active electrodes," *IEEE Trans. Ind. Elect. Contr. Instrum.*, vol. IECI-17, pp. 195–198, 1970.

[2] C. J. Harland, T. D. Clark, and R. J. Prance, "Remote detection of human electroencephalograms using ultrahigh input impedance electric potential sensors," *Applyed Physics Letters*, vol. 81, no. 17, pp. 3285–3286, 2002.

[3] J. Ollikainen, M. Vauhkonen, P. A. Karjalainen, and J. P. Kaipio, "Effects of electrode properties on EEG measurements and a related inverse problem," *Med. Eng. Phys.*, vol. 22, pp. 535–545, 2000.

[4] J. W. H. Meijs, O. W. Weier, M. J. Peters, and A. van Oosterom, "On the numerical accuracy of the boundary element method," *IEEE Trans. Biomed. Eng.*, vol. 36, no. 10, pp. 1038–1049, Oct. 1989.

[5] J. C. de Munck and M. J. Peters, "A fast method to compute surface potentials in the multisphere model," *IEEE Trans. Biomed. Eng.*, vol. 40, no. 11, pp. 1166–1174, Nov. 1993.

[6] C. H. Muravchik and A. Nehorai, "EEG/MEG error bounds for a static dipole source with a realistic head model," *IEEE Trans. Signal Process.*, vol. 49, no. 3, pp. 470–484, 2001.