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# Implicit Dual Snakes for Medical Imaging

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***Abstract*— Dual snake models are powerful techniques for boundary extraction and segmentation of $2D$ medical images. In these methods one contour contracts from outside the target and another one expands from inside as a balanced technique with the ability to reject local minima. Such approach was originally proposed in the context of parametric snakes. Recently, two implicit formulation for dual snakes were presented: our proposal, called the Dual-Level-Set, and the Dual-Front approach. In this paper we review these methods and offer some comparisons. We survey applications for shape recovery in $2D$ cell and human brain MRI images.**

## I. INTRODUCTION

Snake models, originally proposed by Kass at al. [1], are known techniques for boundary extraction and segmentation of medical images. If formulated as a parametric model, they consist of a curve which can dynamically conform to object shapes in response to internal (elastic) forces and external forces (image and constraint ones) [2]. On the other hand, implicit snake models, such as the formulation of *level set* proposed in [3], consist of embedding the snake as the zero level set of a higher dimensional function and to solve the corresponding equation of motion (see [4] for a review).

Parametric models are in general too sensitive to their initial conditions due to nonconvexity problems (see [5] and references therein). To address this limitation some authors have proposed dual methods [6], [7]. The basic idea of the dual active contour models, also called **dual snakes**, is to reject local minima by using two contours: one which contracts from outside the target and one which expands from inside. Such proposal makes possible to reduce the sensitivity to initialization through the comparison between the two contours energy and positions. The two contours are interlinked to provide a driving force to carry the contours out of local minima, which makes the solution less sensitive to the initial position [6].

In [8] we present an implicit formulation for dual snakes, based on the level set approach. The key idea of that work is to view the inner/outer contours as a level set of a suitable embedding function. Besides, a fast dual front implementation of active contours has been proposed in [9]. The method is motivated by minimal path technique [10], [11] and uses fast marching methods [12] to compute the minimal partition curve that represents the object boundary. Dual implicit models have been applied for segmentation of cell and $2D$ human brain MRI images [8], [9]

In this paper we review these models and offer some comparisons between them. In sections II and III we describe the Dual-Level-Set and the Dual-Front techniques, respectively, and present some results obtained with these methods for medical image segmentation. Next, we offer some discussions and comparisons on section IV. In section V we present the conclusions and future developments in this field.

## II. DUAL-LEVEL-SET APPROACH

In this section, we review our work presented in [8]. We maintain the philosophy of dual snakes: one snake contracts from outside the targets and another one expands from inside the target. However, the snake model will be an implicit one which is formulated by the level set approach. The governing equation has the general form:

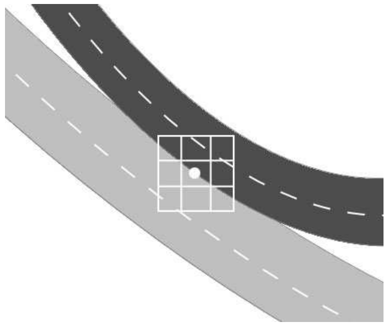
$$G_t = \left( \frac{1 + \alpha k}{1 + |\nabla I|^2} \right) |\nabla G| + \varepsilon \nabla^2 G + \beta \nabla P \cdot \nabla G, \quad (1)$$

where $k$ is the curvature of the front, $\alpha$ weights the curvature dependent speed function, $\varepsilon$ controls a diffusion (viscosity) term that adds stability to the method and $I$ is the image field. The parameter $\beta$ controls the stopping term that is the projection of an attractive force vector, defined through the gradient of the potential $P = -|\nabla I|^2$, on the normal to the front [4]. The function $G$ is an embedding function, which zero level set gives the fronts [8], [13].

The evolution of the fronts follows this governing equation and are interdependent due to the embedding function. The numerical method is a first order one already known in the level set literature [13]. Besides, as usual in level set approaches, we use the narrow band method; that is, only the values of $G$ within a tube placed around the front are updated [3]. For our dual approach, the narrow band is attractive not only for computational aspects but also because it allows an efficient way to evaluate similarity between two contours. In fact, we take the procedure pictured on Figure 1: firstly, the intersection point is computed; then, we take a neighborhood of this point and stop to update the function $G$ in all the grid points inside it or we can set to zero the speed function for these points. We say that those grid points are *frozen* ones.

Once the fronts stop moving, we must decide in which grid points we add a *driving velocity*, called $V_{driv}$. It is an extra velocity term which goal is to keep fronts moving again.



(b)

Fig. 1. (a) Narrow bands touching each other. Neighborhood to define similarity between fronts.

To accomplish this, we use an *affinity operator* [8] which estimates the grid point inside the narrow band most likely to lie away from the target boundaries. Given these points, we apply $V_{driv}$ or simply set $\beta = 0$ for some time steps (see [8] for some definitions of $V_{driv}$).

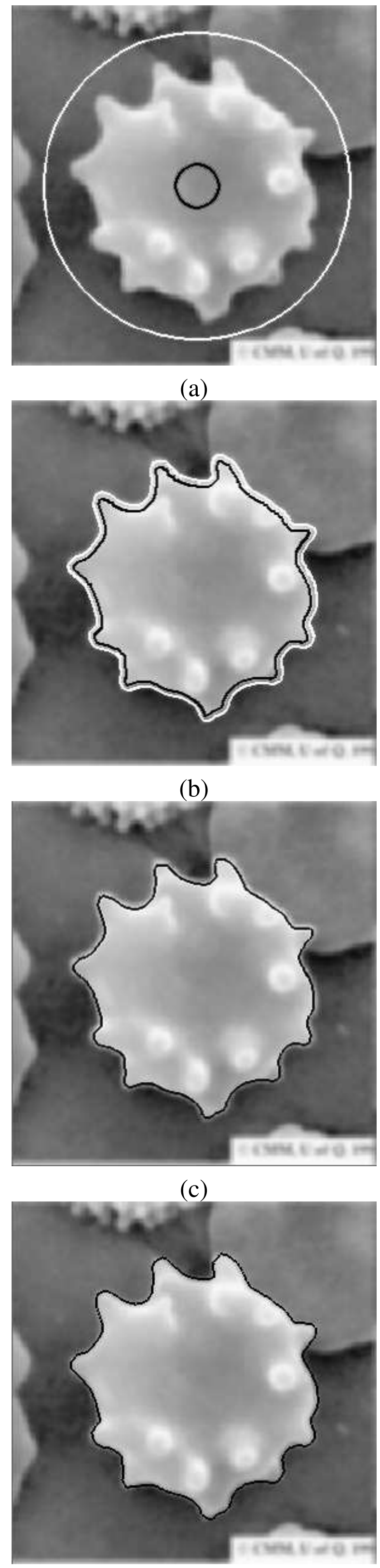
The whole Dual-Level-Set algorithm can be summarized as follows: (1) Initialization; (2) Evolution until fronts stop. (3) Evaluate similarity. If contours are frozen, stop. (4) Add $V_{drive}$ for some time steps. (5) After that, turn-off $V_{drive}$. Go to step 2.

The initialization of the method can be done through signed-distance functions [3] or smoothed step functions [8]. We have applied this technique for medical image segmentation in a two stage approach: (1) the region of interest is reduced by the Dual-Level-Set method; (2) a greedy algorithm (see [8] and references therein) is used to find the object boundaries. Reducing the search space makes the contour detection more reliable and fits very well with dual approaches. Figure 2 shows the application of the Dual-Level-Set to segment the cell image of Figure 2.a. The Dual-Level-Set parameters are: $\alpha = 0.1, \varepsilon = 2.0, \beta = 0.1, T = 150$, and $\Delta t = 0.05$. The Dual-Level-Set result is pictured on Figure 2.b which shows the outer front (white curve) and the inner one (black curve).

This result is used to initialize the next step. Firstly, we compute a curve located in-between the two fronts (Figure 2.c). This curve is the input for a greedy snake which search for the global minimum inside the region bounded by the two fronts in Figure 2.b. Finally, the Figure 2.d shows the result of the greedy method.

## III. DUAL-FRONT APPROACH

In [9], a fast and flexible dual-front implementation of active contours is propose by iteratively dilating an initial curve to form a narrow region and then finding the new closest potential weighted minimal partition curve inside. The method is motivated by minimal path technique [10], [11]. In this method, given a potential $P > 0$, and a point $p$ in the domain $\Omega$, the minimal action map $U_0(p)$ is defined



(a) (b) (c) (d)

Fig. 2. (a) Original image and initialization. (b) Dual-Level-Set result. (c) Initialization of the greedy snake model. (d) Final result.

as:

$$U_0(p) = \min_{A_{p_0,p}} \int_{\Omega} \widetilde{P}(c(s))\, ds \quad (2)$$

where $\widetilde{P} = P + w$, with $w$ been a constant, and $A_{p_0,p}$ is the set of paths connecting $p_0$ and $p$. Expression (2) gives the minimal energy integrated along the paths between the starting point $p_0$ and any point $p$ inside the domain $\Omega$. Because the action map $U_0$ has only one minimum value at the starting point $p_0$ and is a convex function in $\Omega$, it can be easily determined by solving the Eikonal equation [11]:

$$|\nabla U_0| = \widetilde{P}, \quad and \quad U_0(p) = 0, \quad (3)$$

using the fast marching algorithm introduced by Sethian et al. [12]. Equations (2)-(3) are the starting point for the dual-front technique [9]. So, given two points $p_0$, $p_1 \in \Omega$ the method computes the action maps $U_0(p)$ and $U_1(p)$, respectively, through the solution of expression (3), seeking for the points $p \in \Omega$ such that:

$$U_0(p) = U_1(p). \quad (4)$$

At these points, the level sets of the minimal action map $U_0$ meets the level sets of the minimal action map $U_1$ generating the Voronoi diagram of the image that decompose the whole image into two regions containing the points $p_0$ and $p_1$. We can generalize definition (2) for a set $X \subset \Omega$ through the expression:

$$U_X(q) = \min_{p \in X} U_p(q), \quad (5)$$

that mens, $U_X(q)$ is the minimal energy along the paths of the set $A_{p,q}$, where $p \in X$. Therefore, given two curves $c_{in}$ and $c_{out}$ bounding the search space called $R_n$ in the Figure 3, and two potentials $\widetilde{P}_{in}$ and $\widetilde{P}_{out}$ that takes lower values near desired boundaries, the dual-front algorithm firstly computes the minimal action maps $U_{in}$ and $U_{out}$ until these two action maps meet each other. Then, the evolutions of the level sets of both the action maps stops and a minimal partition boundary is formed in the region $R_n$ of the Figure 3. Mathematically, this boundary is the solution of the following equations:

$$|\nabla U_{in}| = \widetilde{P}_{in}, \quad with \quad U_{in}(c_{in}) = 0, \quad (6)$$

$$|\nabla U_{out}| = \widetilde{P}_{out}, \quad with \quad U_{out}(c_{out}) = 0, \quad (7)$$

$$U_{in}(p) = U_{out}(p). \quad (8)$$

The dual-front approach is an iterative method which is picture on Figure 3. Firstly, the curves $C_{in}$ and $C_{out}$ are placed by user interaction or obtained by dilation of an initial curve, named by $C_n$ in Figure 3. Then, the minimal partition boundary is computed by solving equations (6)-(8). To perform this task, the actions $U_{in}$ and $U_{out}$ are computed inside the region $R_n$, through expressions (6) and

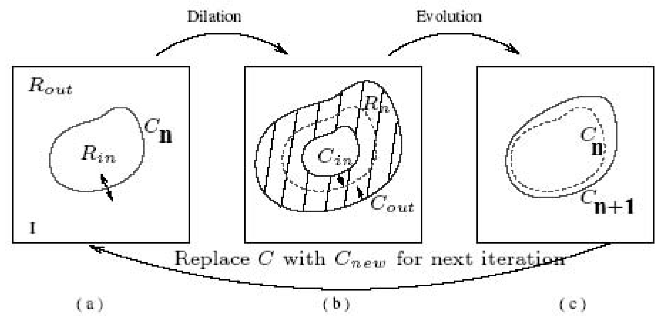



Fig. 3. One interaction of the Dual-Front algorithm. (a)Initial curve $C_n$ of the iteration $n$. (b)Search space defined through dilation of the initial curve $C_n$ with bounds $C_{in}$ and $C_{out}$. (c)Obtained solution $C_{n+1}$, the minimal partition curve. The curve $C_n$ is replaced by the curve $C_{n+1}$ to initialize the next iteration.

(7), respectively, until condition (8) is reached. The obtained result, named by $C_{n+1}$ will replace $C_n$ for processing the next iteration. The method proceed until the distance between consecutive minimal partition curves is less than a pre-defined threshold $\delta$, that means $d(C_n, C_{n+1}) < \delta$. The potentials $\widetilde{P}_{in}$ and $\widetilde{P}_{out}$ are defined in [9] using the following general expressions which integrates region based and the edge-based information:

$$\widetilde{P}_i(x,y) = w_i^r \cdot f\left(|I(x,y) - \mu_i|, \sigma_i^2\right) + w_i^b \cdot g(\nabla I) + w_i, \quad i = in \quad (inner), \quad i = out \quad (outer), \quad (9)$$

where $\mu_{in}$, $\sigma_{in}^2$ are the mean and variance of the image intensity inside region $(R_{in} - R_{in} \cap R_n)$ , $\mu_{out}$, $\sigma_{out}^2$ are the mean and variance of the image intensity inside region $(R_{out} - R_{out} \cap R_{in})$, and $w_{in}^r, w_{in}^b, w_{in}$ are parameters to be set in advance (the same for $w_{out}^r, w_{out}^b, w_{out}$).

The Figure 4 shows an example of the application of the dual-front method for $2D$ human brain MRI images where the segmentation objective is to find the interface between the gray matter and the white matter. This example is interesting to observe the sensitivity of the method against the width of the search space, called active region in [9]. We observe that the obtained result was much better for the narrower search space than for the other ones. In this test, the potentials given by expression (9) are defined by setting $w_0^r = w_1^r = 1$, $w_0^b = w_1^b = 0.1$, $w_0 = w_1 = 0.1$, and $f, g$ given by:

$$f(x,y) = |I(x,y) - \mu_0|, \quad (10)$$

$$g(x,y) = (1 + |\nabla I|)^2. \quad (11)$$

## IV. DISCUSSION

In the Dual-Level-Set one (implicit) snake contracts from outside the target and another one expands from inside the target. In the Dual-Front, actually the level sets of the action map $U$ give the evolution of the front. The velocity of the evolving front is decided by the potentials defined by expression (9) which must be defined such that the velocity

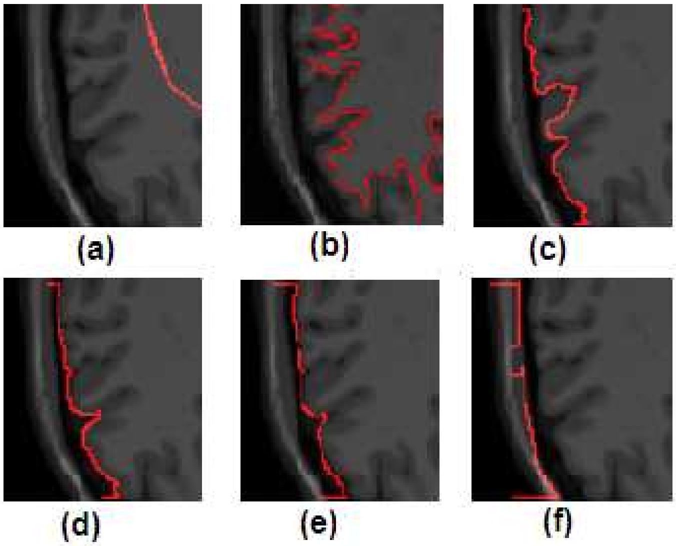



Fig. 4. Sensitivity of the of the Dual-Front against different sizes of the active region (search space): (a) The original $2D$ human brain MRI image and the initial curve; (b)-(f) Segmentation results obtained for a search space defined through morphological dilation of the initial curve with $5 \times 5$, $7 \times 7$, $11 \times 11$, $15 \times 15$, $23 \times 23$ pixels circle structuring elements, after 15 iterations.

is much lower when the evolving fronts arrive the boundary. If the constant $w > 0$, the front velocity will be never null, and so, fronts only stop when the two action maps meet each other. Such policy is interesting to pass over local minima but once there is no an energy balance between fronts the global minimum may be also lost. On the other hand, the critical point for the Dual-Level-Set is to decide the grid points to apply the driving velocity. A bad choice may lead to an undesirable result. Experiments must be performed to compare the efficiency of both Dual-Level-Set and Dual-Front.

Dual-Level-Set is a topologically adaptable deformable model which increases the range of applications. However, such generality has also a price: the care with local minima should be higher than for example, in the original model [6], because there is no a shape model to bias the solution to the desired shape.

Also, we observe in our experiments that sometimes it is more efficient to apply topologically adaptable dual models just to reduce the search space, and, then to apply a search based technique to get the final result, like in Figure 2. It is attractive because it simplifies the choice of parameters for the dual method and makes the computational cost of the application of a global optimization technique smaller.

Despite of the capabilities to reject local minima, dual models have also some disadvantages. Firstly, the method is at least two times more expensive than single approaches. Secondly, the initialization may be a tedious task because the user should set two curves at the beginning of the process. This gets worse if the target is closer to other structures in the image. The choice of parameter values is also another point to be careful because, in this case, there are two snakes to be set. We can use a trial and error procedure or some automatic technique for estimation of parameters (see [14], for example) in order to set one of the snakes, and them apply the opposite of the obtained parameter values for the other one. However, due to inhomogeneities of the background inside and outside the structure of interest, even such method may fail.

## V. CONCLUSIONS AND FUTURE WORKS

Dual approaches are powerful techniques to address the sensitivity to local minima of usual snake models. The idea of using two snakes to seek for the global minimum, originally proposed in [6], has been explored for implicit models through the Dual-Level-Set and the Dual-Front approaches

We must evaluate both the presented implicit dual snakes for some image database (ultrasound, MRI, etc.) in order to compare their efficiency. The precision and computational cost are aspects that we should consider in this point. The automatic initialization of dual approaches is also an interesting problem to be considered in the near future.

## VI. ACKNOWLEDGMENTS

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