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Oscillation and Its Inhibition in A Neural Oscillator Model for Tinnitus

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Abstract— **Tinnitus is a symptom of perceiving phantom sounds. As one of its treatment techniques, tinnitus retraining therapy (TRT) has been proposed. It consists of psychotherapy by counseling and physical therapy based on masking theory by external stimuli. Our interest is to explain medical effects of the physical therapy from the viewpoint of engineering. In this paper we proposed a neural oscillator model with plasticity as a model for the tinnitus generation in the auditory central nervous system and its treatment. We investigated not only oscillatory phenomena observed in the model but also inhibition of the oscillation by external stimulus.**

I. INTRODUCTION

Tinnitus is a symptom of perceiving phantom sounds. The majority of tinnitus cases are caused by misinterpreting null sounds from ears as significant nervous signals in the cerebral limbic system. As a medical treatment for tinnitus, tinnitus retraining therapy (TRT) has been proposed[1], which is based on habituation of the central nervous system. Although it consists of psychotherapy by counseling and physical therapy with external acoustic stimuli, we focus only the physical therapy. In the physical therapy, the cerebral limbic system of a patient is trained with adjusted external sounds so that tinnitus disappers. After that, retrained cerebral limbic system does not perceive tinnitus for a certain period.

In this paper, in order to account for medical effects of the physical therapy in TRT from the viewpoint of engineering, we propose a neural oscillator model with plasticity which has been frequently employed in models of neural systems[2], [3]. To elucidate dynamical properties of the model, we describe a method of analysis[4] based on qualitative dynamical theory, then we investigate bifurcation of oscillation observed in the model. By computing the bifurcations using the method, we can clarify the system parameters contributed to generation of the oscillation and the parameter region in which stable oscillation exists. It is also described that we can inhibit the oscillation by external stimulus and its effective range in the parameters of the stimulus.

II. MODEL DESCRIPTION

We illustrate the architecture of a neural oscillator model in Fig. 1. The model consists of two excitatory neurons, E_1 and E_2, and one inhibitory neuron I. The E_1 and E_2 bidirectionally connect with positive couplings illustrated by white small circles; the E_2 and I also connect with a positive coupling and a negative coupling each other illustrated by the black circle; the S represents external stimulus to the excitatory neuron E_1. This architecture has been employed frequently in modeling neural systems[2], [3]. Its dynamics is described by

$$\frac{dx_1}{dt} \quad (-x_1 + C_{12}Z_2 + S)/\tau_1 \tag{1}$$

$$\frac{dx_2}{dt} \quad (-x_2 + C_{21}Z_1 - C_{2I}Z_i)/\tau_2 \tag{2}$$

$$\frac{dx_I}{dt} \quad (-x_I + C_{I2}Z_2)/\tau_I \tag{3}$$

$$Z_j \quad \frac{2}{\pi} \quad ^{-1} x_j, \tag{4}$$

where x_j, Z_j, and τ_j ($j \quad 1, 2, I$) are the state, the output, and the time constant in each neuron, respectively. The C_{jk} ($k \quad 1, 2, I$) denotes the coupling coefficients from the k-neuron to the j-neuron, which are positive value. Note that only the sign of C_{2I} is negative in (2).

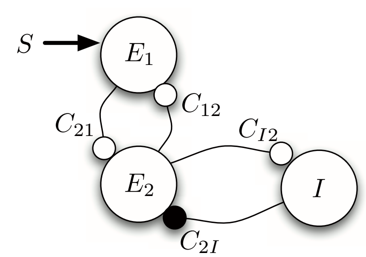

Fig. 1. The architecture of a neural oscillator model is illustrated, which consists of two excitatory neurons, E_1 and E_2, and one inhibitory neuron I. The external stimulation S is added to only the excitatory neuron E_1. The two neurons (E_1, E_2) or (E_2, I) are coupled each other.

III. COMPUTATIONAL METHOD

Oscillations are observed under certain parameter region. By computing bifurcation sets of oscillations, we can investigate the parameter region. The computational method is based on qualitative dynamical theory[4], then we should describe the method before analyzing the model.

Let us consider a general N-dimensional autonomous system which consists of (1)–(4) such that

$$\frac{d\boldsymbol{x}}{dt} \quad f(\boldsymbol{x}), \tag{5}$$

where the state vector $\boldsymbol{x} \in R^N$ corresponds to x_1, x_2, and x_I in the case of the model. We assume that $f(\boldsymbol{x})$ is C^∞-class function for all arguments and there exists a solution

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with initial condition, $\boldsymbol{x} \quad \boldsymbol{x}_0$ at $t \quad t_0$, which is described by $\boldsymbol{x}(t) \quad \varphi(t \ \boldsymbol{x}_0)$ for all t. We also assume that one of the solutions is a limit cycle and its trajectory always intersects with a section in the state space, illustrated in Fig. 2. Thus, arranging a subspace $\quad \subset R^{N-1}$ called Poincaré section, we can define Poincaré map T as

$$T \ \cdot \quad \rightarrow \qquad \boldsymbol{x}_k \mapsto \boldsymbol{x}_{k+1} \quad \varphi(\tau \ \boldsymbol{x}_k). \tag{6}$$

The τ is the time in which the trajectory emanating from a point in $\quad$ at $t \quad t_0$ will across the $\quad$ again. A one-periodic solution in (5) corresponds to a fixed point of T. Hence, an issue for bifurcations of one-periodic solutions returns to an issue for bifurcations of a fixed point of T.

Let $\boldsymbol{x}^* \in \quad$ be a fixed point of T such that

$$\boldsymbol{x}^* - T(\boldsymbol{x}^*) \quad 0. \tag{7}$$

Then its characteristic equation is defined by

$$\chi(\mu) \qquad \left(\mu I_{N-1} - \frac{\partial T(\boldsymbol{x}^*)}{\partial \boldsymbol{x}}\right) \quad 0, \tag{8}$$

where I_{N-1} is the $(N-1) \times (N-1)$ identity matrix. Bifurcation of a fixed point occurs when its topological property is changed by variation of a system parameter, i.e., the characteristic multiplier μ is only on the unit circumference of Gaussian plane. Its types are classified into tangent bifurcation, period-doubling bifurcation, and Neimark-Sacker bifurcation. Then the three bifurcations occur under $\mu \quad 1$, $\mu \quad -1$, and $|\mu| \quad 1$, respectively. Therefore, we can compute bifurcation sets of a fixed point by solving the simultaneous equation composed of (7) and (8).

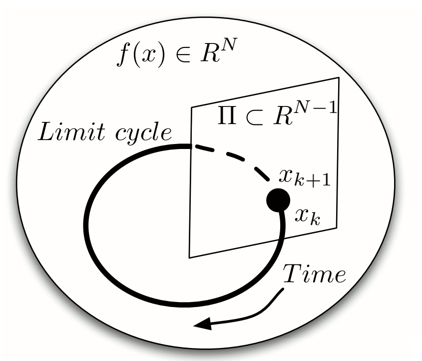

Fig. 2. Poincaré section $\Pi \subset R^{N-1}$ is a subspace of R^N. Poincaré section is arranged so that solutions of $f(\boldsymbol{x})$ across the section.

IV. RESULTS AND DISCUSSIONS

A. Autonomous properties of neural oscillator

In this paper we fixed the parameters:

$$\begin{aligned} &\tau_1 \quad 10, \quad \tau_2 \quad 10, \quad \tau_I \quad 20 \\ &C_{21} \quad 10, \quad C_{I2} \quad 20. \end{aligned}$$

We observed a stationary oscillation under C_{12}=10.0 and C_{2I}=10.0, and its initial state is $\boldsymbol{x}$=(0.1, 0.0, 0.0). The waveforms of the oscillation are shown in Fig. 3. We also observed a stationary equilibrium under the same parameters, but its initial state is $\boldsymbol{x}$=(0.0, 0.0, 0.0). This equilibrium is in the origin, see Fig. 4. Hence, the stable oscillation and the stable equilibrium coexist under the same parameters.

In order to investigate the existing region of the oscillation in the C_{12}–C_{2I} plane, we computed the bifurcation sets of the oscillation. The bifurcation diagram is illustrated in Fig. 5. The curves indexed by G_o denotes the tangent bifurcation of the oscillation, then the oscillation exists in only the region surrounded by the two curves. Besides, the equilibrium exists in all region of the same figure. That is, one stable oscillation and one stable equilibrium coexist in the region surrounded by G_o, then the oscillation observed in such bistable state is called stiff oscillation.

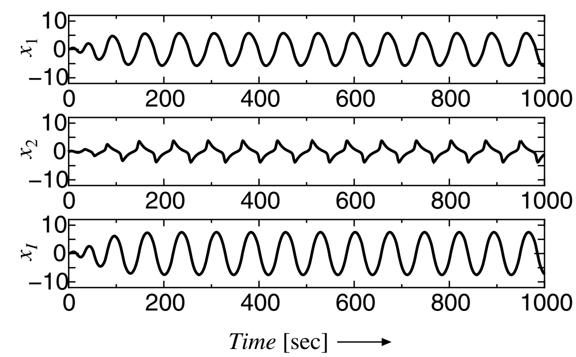

Fig. 3. The oscillation can be observed under C_{12}=10.0 and C_{2I}=10.0. Its initial state is $\boldsymbol{x}$=(0.1, 0.0, 0.0).

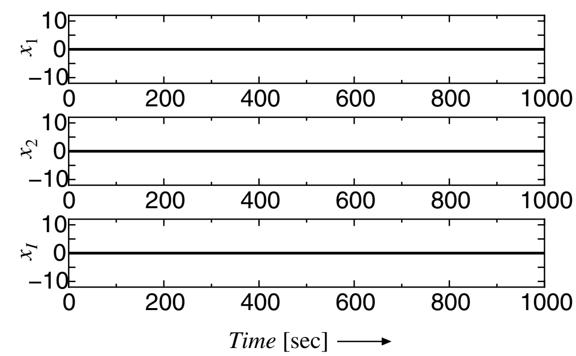

Fig. 4. The equilibrium can be observed under C_{12}=10.0 and C_{2I}=10.0. Its initial state is $\boldsymbol{x}$=(0.0, 0.0, 0.0).

B. Properties of plastic neural oscillator

Let us consider a plastic neural model, e.g., the coupling C_{12} has plasticity defined by

$$\frac{dC_{12}}{dt} \quad \left(-C_{12} + bZ_1Z_2 + C_0\right)/\tau_c, \tag{9}$$

where b is the coupling parameter of E_1 and E_2, then C_0 is the bias of C_{12}. Hence, the dynamical system of the plastic neural oscillator model is described by (1)–(4) and (9). Because we found the lower limit value of C_{2I}, that is the line indexed by L_o, in Fig. 5 that the oscillation can be observed, we fixed the parameter as C_{2I}=10. Note that

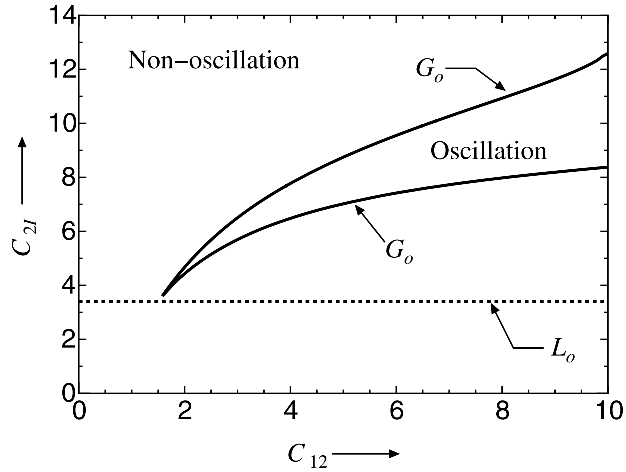

Fig. 5. This is bifurcation diagram of the oscillation shown in Fig. 3 on C_{12}–C_{2I} plane. The curves indexed by G_o indicate tangent bifurcation of the oscillation.

its lower limit of C_{12} is given by the nodical coordinate between the upper curve of G_0 and the line with C_{2I}=10, that is C_{12}=6.618. We also fixed the other parameters as b=20 and τ_c=500. Figure 6 shows the stationary values of C_{12} by changing the value of C_0. The lower line indicates that the stationary state converges to the stable equilibrium, then the upper curve denotes that it converges to the stable oscillation. Then the oscillation and the equilibrium coexist in the range of C_0=[2.65, 8.06], which is interleaved by the two dotted lines. From an engineering viewpoint we are interested in the region of system parameters that the stiff oscillation is observed and whether the oscillation can be inhibited by an external stimulus.

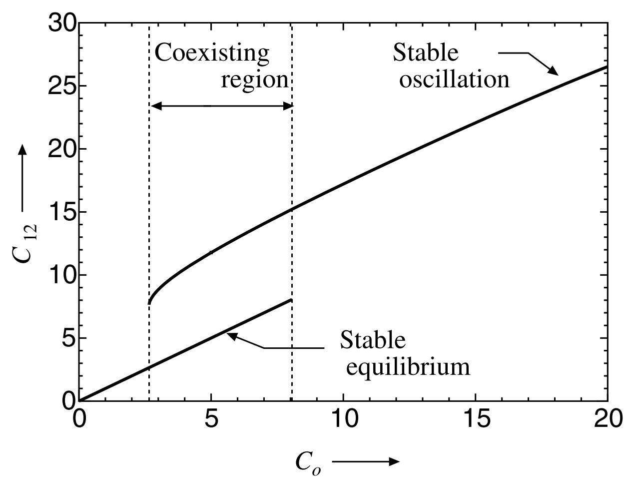

Fig. 6. This shows the stationary value of C_{12} with any initial value for each C_0. The parameters are fixed as C_{2I}=10, b=20, and τ_c=500. The upper curve and the lower line denote existence of equilibrium and oscillation, respectively.

C. Inhibition of oscillation

We fixed the parameter as C_0=5 so that the stiff oscillation appears, see Fig. 6. Let us consider inhibition of the oscillation by external stimuli: the sinusoidal stimulus defined by

$$S \quad V_e \quad 2\pi f_e t \tag{10}$$

and DC stimulus described by

$$S \quad V_d. \tag{11}$$

By applying the sinusoidal stimulus to E_1, the stationary value of the plastic coupling C_{12} changes. We show the phenomenon under V_e=2 in Fig. 7. The dotted line indicates the threshold whether stable oscillation occurs or the oscillation is inhibited; its value of C_{12} is 6.618 as mentioned above. The stationary value of C_{12} with the initial state $C_{12}(0)$=11.8 is under the threshold line in the range of f_e=(0.0, 0.011], then it is above the line otherwise. The initial state $C_{12}(0)$=11.8 means the parameter value when the stationary oscillation appears. Hence, we can inhibit the oscillation by adding such sinusoidal stimulus with V_e=2 and f_e=(0.0, 0.011]. Then we investigated the parameter region in f_e–V_e plane that we can inhibit the oscillation; and its region is shown in Fig. 8. If we fixed the two parameters V_e and f_e in the black region, the oscillation can be inhibited. We are interested in the followings: we can inhibit the oscillation only around V_e=2, which is smaller than the amplitude of x_1; and we can also inhibit the oscillation only if f_e is similar in the frequency of x_1. We showed simulation results for two sinusoidal stimuli with different frequency in Figs. 9 and 10. By applying a sinusoidal stimulus with similar frequency to x_1, the value of C_{12} slowly decreases under the threshold line of occurrence of oscillation. After that the oscillation disappears without the stimulus. Withal, the sinusoidal stimulus with high frequency cannot reduce the value of C_{12}, then the oscillation goes on.

We also investigated inhibition of the oscillation for DC stimulus. As shown in Fig. 11 we can inhibit the oscillation by DC stimulus, however, its parameter region of V_d is restricted in around [1.2, 1.6].

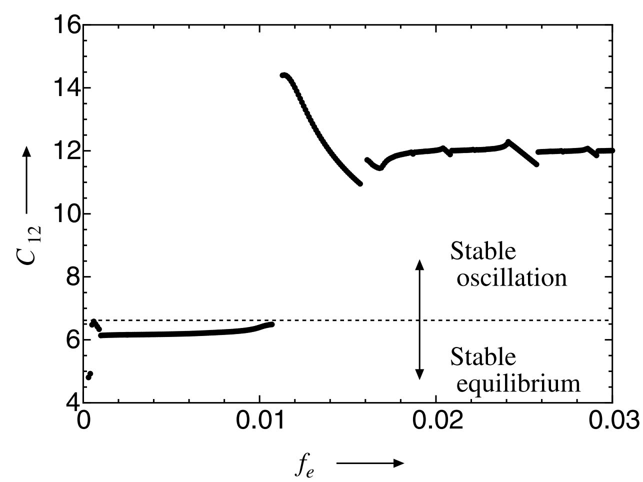

Fig. 7. The figure is one-parameter bifurcation diagram of C_{12} for f_e under V_e=2. It indicates stationary value of C_{12} under each f_e. Its initial value is fixed as $C_{12}(0)$=11.8. The dotted line indicates the threshold whether stable oscillation occurs or the oscillation is inhibited.

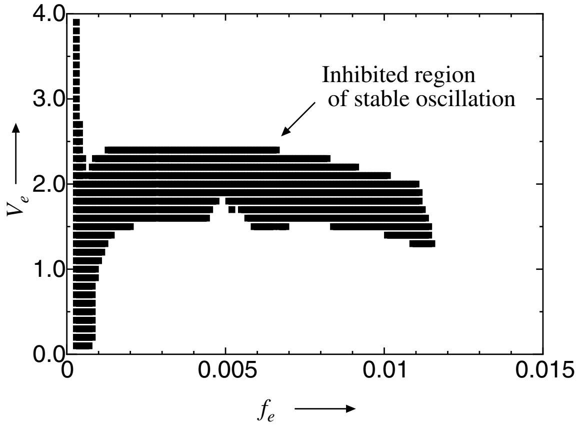

Fig. 8. This is the basin of the stable equilibrium under external periodic stimulus described by (10). That is, we can inhibit the stable oscillation in the black region of f_e–V_e plane.

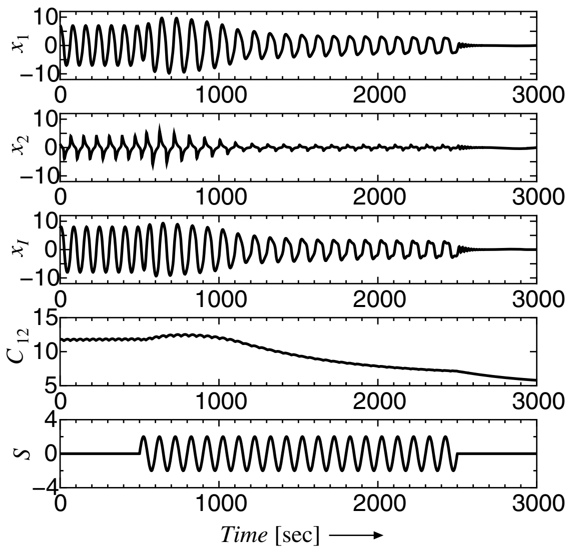

Fig. 9. These waveforms are simulation results by adding the sinusoidal stimulus: $S = 2 \sin 0.02\pi t$. The external stimulus is applied in only from 500 to 2500 sec.

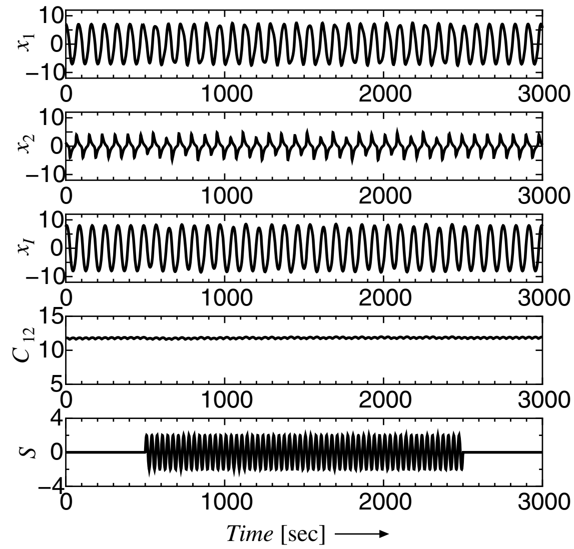

Fig. 10. These waveforms are simulation results by adding the sinusoidal stimulus: $S = 2 \sin 0.03\pi t$. The external stimulus is applied in only from 500 to 2500 sec.

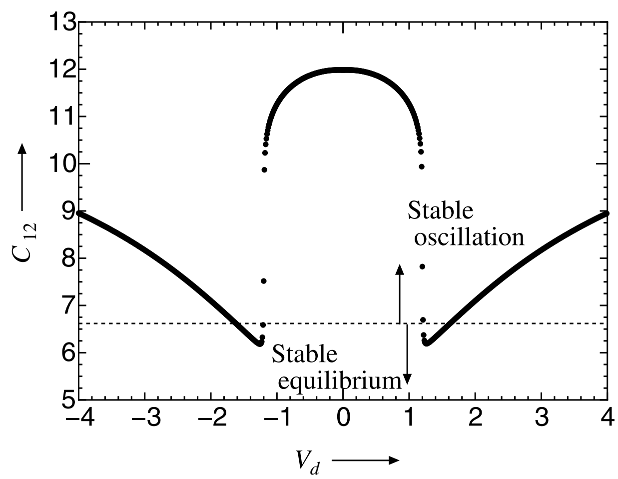

Fig. 11. The figure is one-parameter bifurcation diagram of C_{12} for the DC stimulus described by (11). It denotes stationary value of C_{12} under each V_d, and its initial value is fixed as $C_{12}(0)$=11.8. The dotted line indicates the threshold whether stable oscillation appears; the value of C_{12} is 6.618.

V. Concluding Remarks

To account for medical effect of TRT from the viewpoint of engineering, we proposed a neural oscillator model with plasticity. We investigated the autonomous properties of the neural oscillator model, then it was discovered that stiff oscillation appears in a certain parameter region. We also considered inhibition of the oscillation in bistable state using two kinds of external stimuli: sinusoidal stimulus and DC stimulus. It consequently was found out that we can inhibit the oscillation only under certain range of amplitude and frequency. If it assumed that the oscillation and the equilibrium respectively correspond to occurrence of tinnitus and disappearance of tinnitus, we can explain the fundamental mechanism of generation of tinnitus and treatment of tinnitus by external stimulus.

In our future works, to clarify the mechanism of inhibition of the oscillation in the model, we should investigate bifurcation phenomena of the oscillation observed in the dynamical system with some kinds of external stimuli.

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