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The Use of Genetic Algorithms for Solving the Inverse Problem of Electrocardiography

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Abstract—**Reconstruction of the epicardial potentials from the body surface potentials constitutes one form of the ill-posed inverse problem of electrocardiography (ECG). In this paper, we investigate the use of genetic algorithms (GAS) for regularizing ill-posed ECG inverse problem. The result shows that, GAS cannot be used to regularized ill-posed problem without additional constraints, but combined with other methods or additional information about solutions, GAS is an efficient optimization technique for solving the ill-posed inverse problem. We adopt the Tikhonov regularized solutions as the additional information to construct the initial populations. This investigation suggests that the GAS may provide a useful tool for ECG inverse problem studies.**

Keywords—**genetic algorithms, inverse problem, Tikhonov regularization, discrepancy principle**

I. Introduction

The inverse problem of electrocardiography (ECG) has as its goal the reconstruction of cardiac electrical events from information obtained noninvasively at the body surface. At each instant of time throughout the cardiac cycle, the potential distribution over the entire torso constitutes the complete set of data in noninvasive electrocardiography. The ability to noninvasively relate surface potential patterns to regional cardiac events is of great physiological and clinical importance, and epicardial potentials accurately mirror details of the electrical events within the myocardium with high resolution. The problem of noninvasively computing the potentials on the heart's epicardial surface from surface potentials recorded on the body constitutes one form of the inverse problem of ECG [1]. With the source determined, our specific inverse problem is to describe potentials on the epicardial surface as a function of those on the surface of the thorax, together with the knowledge of the geometric and resistive features of the intervening volume conductor. To describe the relationships mathematically, we use the cardiac sources in terms of epicardial surface potentials, and solve a generalized Laplace's equation with Cauchy boundary conditions:

$$\begin{cases} \nabla \cdot \delta \nabla \Phi = 0 \quad in \ \Omega \\ \delta \nabla \Phi \cdot n = 0 \quad on \ \Gamma_T \quad find \ \Phi_E \text{ on } \Gamma_T \\ \Phi = \Phi_B \quad on \ \Gamma_T \end{cases} \qquad (1)$$

where Φ are the electrostatic potentials, δ is the conductivity tensor, Γ_T and Ω represent the surface and the volume of the thorax, *and* Γ_E is the surface of epicedium. Boundary element method (BEM) is used to seek the transfer matrix A:

$$\mathbf{\Phi_B = A\Phi_E} \qquad (2)$$

However, the A gotten by the BEM is ill-posed in that small measurement errors in the surface potentials, or any geometry error in the volume conductor model used in the inversion procedure, lead to large perturbations in the epicardial distributions. This renders a conventional least square error inverse, characterized by the minimization of the functional:

$$\boldsymbol{M}(\phi) = \min \left\| A\Phi_E - \Phi_B \right\|_2 \qquad (3)$$

We note that many of the inverse problems encountered in engineering may be reformulated as optimization problems but often the corresponding object function may be highly nonlinear or nonmonotonic, may have a very complex form or its analytical expression may be unknown. Genetic algorithms do not require knowledge of the gradient of the object functions, which make them particularly suitable for optimization problems for which an analytical expression of the object function is unknown. Mera et al has done some research on the use of genetic algorithms for solving ill-posed problem [2]. The use of genetic algorithms in ECG inverse problem, however, has not been reported yet. Thus the purpose of this study is to investigate how to use the genetic algorithms for solving the ill-posed ECG inverse problems.

II. Methodology

A. Genetic Algorithms

Genetic algorithms are stochastic optimization techniques that attempt to model the process of natural evolution, the survival of the fittest individual and the strive for survival. A genetic algorithm performs a multi-directional search. It starts with a randomly initialized population of candidate solutions and implements a probabilistic, parallel search in the solution space to form a new population of candidate solutions. The population undergoes a simulated evolution process. At each generation the relatively "good" solutions reproduce, while the relatively "bad" solutions die. To distinguish between different solutions we use a fitness (evaluation) function which plays the role of an environment [3, 4]. In order to solve the ECG inverse problem by applying genetic algorithms, we consider float number encoded genetic algorithm which is using tournament selection, scatter crossover, uniform mutation, and elitist selection. The genetic operators and the parameters used for the genetic algorithms are taken to be

- Population size $n_{pop} = 200$
- Scatter crossover
- Tournament selection, tournament size $k = 2$, tournament probability $p_t = 0.8$

- Uniform mutation, mutation probability $p_m = 0.02$
- Elitism, elitism parameter $n_e = 2$

The offspring is produced from the current population using the above listed genetic operators. Next, the parents' population and the children population is merged and using tournament selection, n_{pop} individuals are selected for survival to the next generation.

The ill-posed problems of the form (2) considered in this paper were reformulated as optimization problem by minimizing the object function, such as the formulation of (3).

The number of generations to be performed cannot be a priori determined, but it is determined during the evolution by using the discrepancy principle.

B. Tikhonov Regularization Methods

A common approach used to overcome the instability of the solution is via zero-order Tikhonov regularization [5, 6], in which instead of the functional in (3), the alternative functional (4)

$$M(\phi_E) = \min_{V_E} \|\phi_B - A\phi_E\|^2 + \gamma \|\phi_E\|^2 \quad (4)$$

is minimized. Here, the squared norms of both the surface potential residual and the solution are minimized together, the idea being to suppress unwanted oscillations in the solution.

The corresponding solution is

$$\phi_E = (A^T A + \gamma I)^{-1} A^T \phi_B \quad (5)$$

where I is an $n \times n$ identity matrix and the parameter ($\gamma > 0$), known as the regularization parameter, serves to determine the relative weight accorded to each of the two terms in (4). A different solution, ϕ_E, results for each choice of γ. When the epicardial distribution is known ahead of time (as occurs in simulation studies), it is possible to gauge the correctness of the solution by either relative error (RE), namely

$$\boldsymbol{RE}(t) = \frac{\|\Phi_t - \Phi_E\|}{\|\Phi_E\|} \quad (6)$$

or the correlation coefficient (CC), given by

$$\boldsymbol{CC}(t) = \frac{\sum_{i=1}^{n} [(\Phi_t)_i - \bar{\Phi}_t][(\Phi_E)_i - \bar{\Phi}_E]}{\|\Phi_t - \bar{\Phi}_t\| \|\Phi_E - \bar{\Phi}_E\|} \quad (7)$$

In (6) and (7), Φ_E denotes the known epicardial distribution, and Φ_t the computed one. The quantities $\bar{\Phi}_E$ and $\bar{\Phi}_t$ are, respectively, the mean values of Φ_E and Φ_t, over the n epicardial sites.

There exist different ways of choosing the regularization parameter. Here we employ the L-curve technique [6, 7]. The L-curve approach involves a plot using a log-log scale of the norm of the solution $\|\phi_E\|$ on the ordinate, against the norm of the residual $\|A\phi_E - \phi_B\|$, on the abscissa, with γ as a parameter along the resulting curve. As long as the uncorrelated Gaussian noise present in ϕ_B dominates the correlated geometric noise in *A,* this curve is in the form of an "L" and the value, γ, at the corner of the "*L*" is selected. At the corner, both $\|A\phi_E - \phi_B\|$ and $\|\phi_E\|$ simultaneously attain low values, intuitively suggesting a reasonable solution. A numerical algorithm to compute the site of the corner as the point of maximum curvature can be found in Hansen [7].

C. Discrepancy Principle

Discrepancy principle [8], in all simplicity, amounts to choosing regularization parameter such that the residual norms for the regularized solution satisfy:

$$\|A\phi_E - \phi_B\|_2 \approx \|e\|_2 \quad (8)$$

where $\|e\|_2$ is an estimate of noise present in the inverse problem, i.e., $\|e\|_2 = \|\hat{\phi}_B - \phi_B\|_2$, $\hat{\phi}_B$ is the exact potential without noise, and ϕ_B is potential with arbitrary small noise level $\|e\|_2$.

In this paper, the discrepancy principle may be used for choosing the generations by stopping the reproduction process when

$$\|Ax_k - \phi_B\|_2 \approx \|e\|_2 \quad (9)$$

where x_k is the numerical solution obtained after k reproductions.

D. Simulation Protocol

The simulation protocol is used to test the validity of genetic algorithms for solving the ECG inverse problem. We adopt a concentric sphere model to represent the heart and body surfaces, with known analytical distributions on both being induced by placing a current dipole within center of the inner sphere. In this way, 2*cos* (*theta*) *mv* potential distribution on the inner surface is produced, and here *theta* is the angle with Z-axis. To be somewhat representative of the epicardium and human torso, the radii of the inner and the outer spheres are chosen as 4 *cm* and 10 *cm*, respectively [9]. The inner and outer spheres are discretized using uniform spherical coordinate grids consisting of 120 points and 190 points, respectively. Two additional grid points are added, one at the top and the other at the bottom, so that the resulting triangularized surfaces are closed, as shown in Fig. 1. The outer surface comprises 192 nodes and 380 triangles, and there are 122 nodes and 240 triangles in the inner surface. The boundary element method, based on a triangular mesh with a linear variation in potential [10], is used to derive the 192 × 122 transfer matrix A, which is adopted in the inverse computation. Next, outer surface potentials at its 192 surface nodes are computed from these 122 inner surface potentials using the 192 × 122 A matrix, determined again by the boundary element method. In the inverse direction, however, potentials are sought at 122 uniformly distributed inner points, to be computed from 192 outer surface sites distributed, and an additional 10 *dB* white Gaussian noise $\boldsymbol{e}$ is added to the outer surface potentials, where $\|e\|_2$ equals to 1.8955.

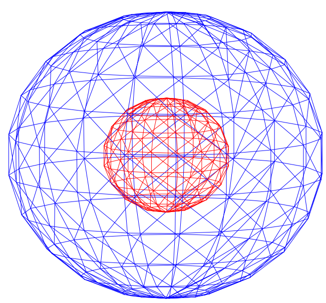

Fig. 1. The triangularized surfaces of the concentric spheres model, the outer surface comprised 192 nodes and 380 triangles, the inner surface consisting of 122 nodes and 240 triangles.

III. Results

From Fig. 2, we can see that after only 20 iterations the fittest individuals in the population give a very good fit to the data which satisfy the discrepancy principle ($\boldsymbol{M}(\phi) = 1.9242 \approx \|\boldsymbol{e}\|_2$), but the solutions of the ill-posed are not exact which attain a high relative error, (RE=1.0264, CC=0.7122). Once such an unstable solution is generated in the genetic algorithm, since it gives a better fit to the data than the exact solution, then it will prevent the exact solutions and the solutions close to it from being considered solutions to the problem as the fittest individuals of a generation. Furthermore, by providing a very good fit to the data, then solutions have a very high fitness value and therefore may force the genetic algorithm search to focus around it or similar incorrect solutions. As shown in Table1, after 200 iterations, we still get the "false" solutions with RE= 0.9347, CC= 0.7491. Moreover, since the initial population and some of the individuals created during the early generations are randomly generated, the global optimum of the problem, which is a highly unstable and a non-smooth solution could appear in the population at an early stage causing the genetic algorithm to be stationary from that point onward. However, for ill-posed problems the solution to the problem is not the global optimum of the fitness function but a solution to satisfy the optimum balance between the stability (smoothness) and the fit to the data. Therefore in the approach considered here genetic algorithms cannot be used to regularize ill-posed problems since they may produce highly unstable solutions at the very first generations and then they may stagnate around these solutions.

However, if additional information about the solution of the problem is available, then such information should be used to add constraints that can prevent unstable solutions from being generated within the genetic algorithm. Once such constraints have been added, a genetic algorithm can be successfully applied to find the global optimum of the problem and will get good solution with low relative error and high correlation coefficient. Here we use the Tikhonov regularization method to find the information about the solution of the inverse problem, and use the regularized solution to construct the initial populations which can be acted as the constraints. After adding such constrains, we can get relative good solutions with RE= 0.2276, CC= 0.9739, and $\boldsymbol{M}(\phi) = 1.8752 \approx \|\boldsymbol{e}\|_2$, as shown in Fig. 3.

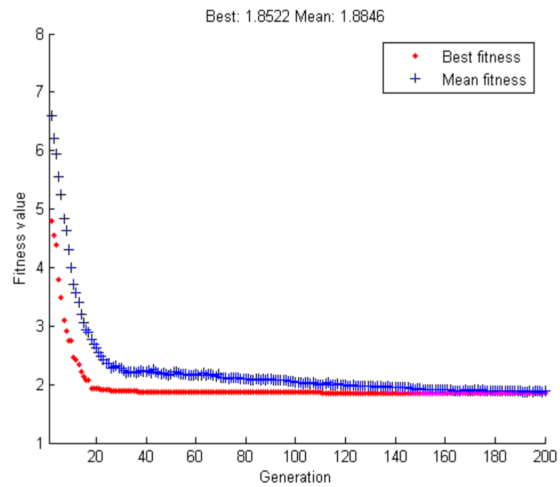

Fig. 2. The error *M(ϕ)* as function of the number of generations performed, *M(ϕ)* on the ordinate and generations on the abscissa.

Table.1 Comparison of the object function *M(ϕ)*, *RE*, and *CC* on different generations

Generations	***M(ϕ)***	***RE***	***CC***
20	1.9242	1.0264	0.7122
50	1.8721	0.9155	0.7452
100	1.8654	1.0394	0.6994
200	1.8491	0.9347	0.7491

IV. Discussion And Conclusion

The use of the genetic algorithms for solving ill-posed ECG inverse problem has been investigated. The results show that genetic algorithms do not have a regularizing character and therefore they cannot be used to regularize an ill-posed problem if the ill posedness is not dealt with by another method. However, it was found that GAS could be applied to solve the ill-posed inverse problem if additional information about the solutions or other additional constraints is available. Those additional information and constraints can prevent the false solutions from being generated within the genetic algorithms. How to find the available information and constrains plays a key role in using the GAS for solving the ill-posed inverse problem. In this paper, we adopt the Tikhonov regularized solutions as the additional information about solutions, and the results shows that we can get the stable solutions with low relative error and high correlation coefficient. Therefore, GAS may be a useful tool for solving ECG inverse problem.

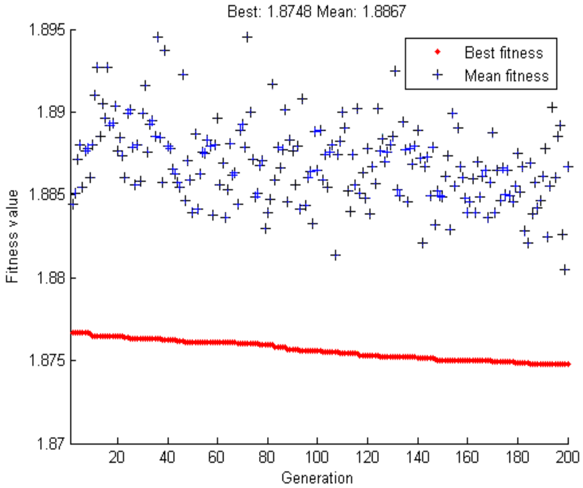

Fig. 3. The error $M(\phi)$ as function of the number of generations performed, $M(\phi)$ on the ordinate and generations on the abscissa. The initial population is made up from the regularized methods.

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