Tissue Image Interpolation Based on Fractional Brownian Motion

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Abstract—**Fractal Brownian Motion (fBm) has been proven in a viewpoint of irregularity characterization for quantifying the structure of the tissue images. Breast masses were selected as the target sample because breast-associated diseases have become prevalent in Taiwan. In this paper, the ability of fractal dimension in fractal interpolation schemes is investigated and compared with the traditional interpolation schemes including bilinear and bicubic methods. Using three image quality indices the difference between original and interpolated images may have evaluated. Interpolated images by fractal interpolation can maintain fractal characteristics better than traditional interpolation methods. Fractal features can be preserved in the interpolated images. Thus, interpolated tissue images based on fBm model are superior to conventional methods.**

I. Introduction

Fractals are widely existed around the world including the geographical landform, plants and creatures. Properties of fractals are represented in parts, chips, and self-similarity, providing a method to describe complex objects, such images from tissues. Traditional interpolation methods [1] for images highly rely on linear and high order polynomial based on the assumption of high-order continuity. The spline interpolation, nearest interpolation, bilinear interpolation, cubic convolution [2], local lagrange, and tapered sinc are focued on nearby continuity, but global features may not be considered in these methods. Unfortunately, tissue image has large portion of discontinuity and edges. In this situation, fractal interpolation becomes suitable. It is known that fBm [3] can provide a fine description corresponding to surface existed in nature environment.

In order for assessing the interpolation performance, three indices such as fractal index H, peak signal-to-noise ratio(PSNR) and new quality index(NQI)[4] are used to evaluate quality of interpolated images. Fractal index H can measure an interpolated image that preserves ability of fractal dimension. Therefore, fractal dimension of our fBm processed tissue images approximated original images. According to fractal index H, fractal interpolation method is better than traditional ones. In the aspect of PSNR, fractal interpolation was close to bilinear approaching and better than bicubic method. Finally bicubic method is superior to other methods by NQI.

II. Interpolation Methods

There are many traditional interpolation methods which assume the continuity in space domain. Two of traditional interpolation methods including bilinear and bicubic methods are employed for comparison.

A. Bilinear interpolation

The values of both direct neighbors that are weighted by their distance to the opposite point of interpolation in separated bilinear interpolation. Therefore, the bilinear interpolation is approximation of sinc function by using linear triangle function

$$W(x) \cong \begin{cases} 1-|x|, & 0 \le |x| < 1 \\ 0, & \text{elsewhere} \end{cases} \quad (1)$$

Bilinear interpolation is represented as equation (2) and Fig. 1.

$$Z(x_p, y_p) = (1-dx)(1-dy)Z(1,1) + dx(1-dy)Z(1,2) + (1-dx)dyZ(2,1) + dxdyZ(2,2) \quad (2)$$

where $Z(x_p, y_p)$ is gray value of the interpolated image.

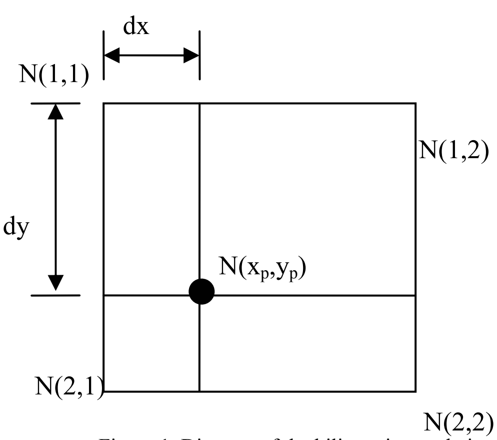


Figure 1. Diagram of the bilinear interpolation

B. Bicubic interpolation

Cubic polynomial is frequently used and the symmetric kernel is defined as

$$V(x) = \frac{\sin \pi x}{\pi} = \begin{cases} (a+2)|x|^3 - (a+3)|x|^2 + 1, & 0 \le |x| < 1 \\ a|x|^3 - 5a|x|^2 + 8a|x| - 4a, & 1 \le |x| < 2 \\ 0, & \text{elsewhere} \end{cases} \quad (3)$$

$$Where \quad a = -1$$

The bicubic interpolation is approximation of sinc function by using third order polynomial function.

C. Fractional Brownian interpolation

This model is an adaptive manner about Random Midpoint Displacement technique [5][6]. According to fBm model, the expected value of the difference in elevation over the distance △x is proportional to $(\triangle x)^H$

$$\log E[|z(x+\Delta x)-z(x)|]=H\log\Delta x+K \quad (4)$$

Where both H and K are constant, Parameter H can represent equation (5) with fractal dimension:

$$D=3-H \quad (5)$$

Parameter D approaching to constant 2 means high correlation with adjacent points in plane. In contrast, Parameter D approaching to constant 3, it has high complexity in curved surface.

According to equation (4), the fBm index is computed from the slope of the expected value plotted as a function of △x in the log-log plot in linear regression and its slope is H.

The distance(△x) of input image with one is used to obtain variance σ^2 of the difference of gray value. The variance $\sigma_{1/2}^2$ over distance △x equal to 1/2 pixel could be obtained with equation (6)

$$\sigma_{1/2}^2=(\frac{1}{2})^{2H}\sigma^2 \quad (6)$$

Next, let σ_b^2 be the variance of the difference in bilinear interpolation following equation (7)

$$\sigma_b^2=(\frac{1}{2})^2\sigma^2 \quad (7)$$

The fractal dimension is 2 in bilinear interpolation about H=1.

The weighted Brownian interpolation which adds random term to the bilinear interpolation was proposed by Polidori[5]. In his model, when △x equal to 1/2, the variance is the difference between σ_b^2 and $\sigma_{1/2}^2$. Following equation (8)

$$\eta^2=(\frac{1}{2})^{2H}\sigma^2-\frac{\sigma^2}{4} \quad (8)$$

Thus,

$$\eta=\frac{\sigma}{2^H}\sqrt{1-2^{2H-2}} \quad (9)$$

So, the fractal interpolation may be obtained as equation (10).

$$Z(i,j)=Z_b(i,j)+\eta GC \quad (10)$$

where G is Gaussian random distribution, and C is correlation coefficient between △x and $E[|Z(x+\Delta x)-Z(x)|]$ in log-log plot. It responses fractal characteristic of different range of input image. $Z(i,j)$ is gray value of the interpolated image by fractal interpolation, $Z_b(i,j)$ is gray value of the interpolated image by bilinear interpolation.

III. Image Quality Index

After interpolating, the image is assessed using the quality indices including fractal index H, PSNR, and NQI.

A. Fractal index H

Fractal dimension is an interesting feature to characterize roughness and self-similarity of an image. According to equation (4), H is obtained as the slope of the expected values verses a function of △x in the log-log plot in a linear regression..

B. Peak signal to noise ratio

PSNR is a popular measure to compare the difference between the original and interpolated images. PSNR is defined as

$$PSNR=10*\log(\frac{255^2}{MSE})$$
$$MSE=\frac{\sum_{n=1}^{Image\ Size}(I_n-P_n)^2}{Image\ Size} \quad (11)$$

The maximum value equals to 255 for 8 bits of gray levels. MSE stands for the mean square error. The nth pixel value in original image is denoted as I_n. P_n is the Nth pixel value in processed image. The unit of PSNR is dB. The greater PSNR value, the less the distortion of interpolated image will be. As a kind of evaluation parameters, PSNR can not stand for human experiences in certain situation. For example, different averaged luminance of the same image can greatly affect PSNR values. However, human vision hardly regards the averaged luminance difference as an important factor in image recognition.

C. New quality index

The new quality index is defined as

$$Q=\frac{4\sigma_{xy}\bar{x}\,\bar{y}}{(\sigma_x^2+\sigma_y^2)\left[(\bar{x})^2+(\bar{y})^2\right]}=\frac{\sigma_{xy}}{\sigma_x\sigma_y}\cdot\frac{2\bar{x}\,\bar{y}}{(\bar{x})^2+(\bar{y})^2}\cdot\frac{2\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2} \quad (12)$$

where,

$$\bar{x}=\frac{1}{N}\sum_{i=1}^{N}x_i,\quad \bar{y}=\frac{1}{N}\sum_{i=1}^{N}y_i$$
$$\sigma_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2,\quad \sigma_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{y})^2$$
$$\sigma_{xy}=\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})$$

The first component is the correlation coefficient between x and y. The second component is to measure how close the mean luminance between x and y. Finally, the third component is to measure how similar the contrasts between original image and interpolated image. So the image quality index (NQI) is useful to measure any image distortion, the proposed index is designed as a combination of three factors: loss of correlation, luminance distortion, and contrast distortion.

IV. Experiment and Results

Fig. 2 shows the procedure of image processing. First of all, two tissue images (256*256) are acquired. Secondly, the Nth pixel for every N pixel by decimation is obtained [7]. Thirdly, η, H *and C* are computed according to equation (9). Using equation (10), images may be interpolated with random value ranging from -0.3 to 0.3. Finally, interpolated images are evaluated by three image quality indice such as fractal index H, PSNR and NQI.

Original image is shown in Fig. 3(a). The interpolated images are similar, because fractal interpolation method is based on bilinear method. Finally, the results for bicubic interpolation method are shown in Fig. 3(d). The visual effect of image smoothness is increased so as to blur edges and textures.

In Table 1, the values for three image quality indices for Fig. 3 are listed for comparison. Similarly, Table 2 lists the data for Fig. 4.

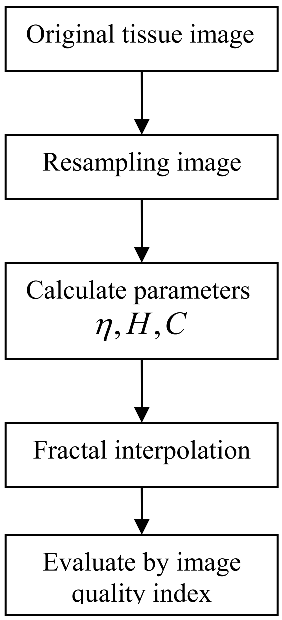
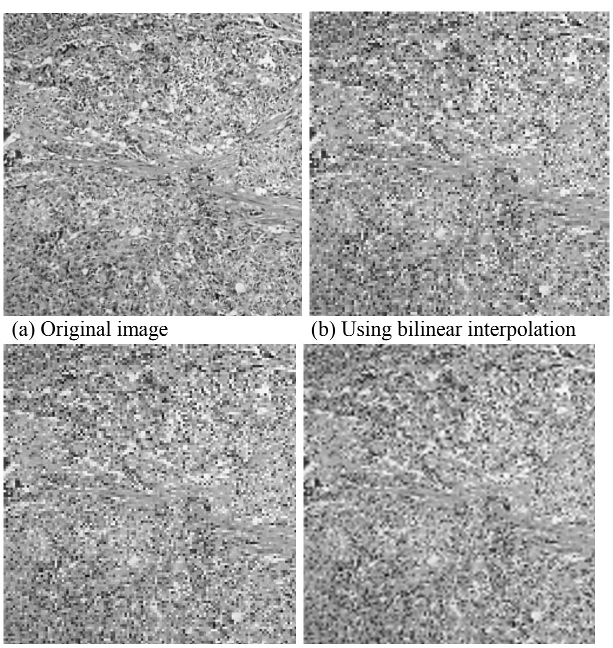


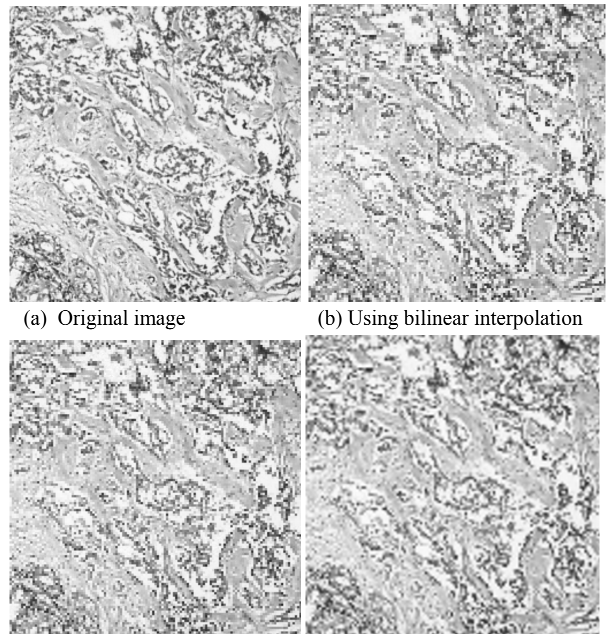
Figure 2. Flowchart of the processing procedure



(a) Original image (b) Using bilinear interpolation

(c) Using fractal interpolation (d) Using bicubic interpolation

Figure 3. Interpolated tissue1 image



(a) Original image (b) Using bilinear interpolation

(c) Using fractal interpolation (d) Using bicubic interpolation

Figure 4. Interpolated tissue2 image

Table 1. The result of the quality index for tissue1 image

	Fractal Index H	PSNR	NQI
Original	0.1229	-	-
Fractal	**0.1965**	**68.1298**	**0.9988**
Bilinear	0.2245	68.2133	0.9988
Bicubic	0.2185	66.3281	0.9988

Table 2. The results of the quality index for tissue2 image

	Fractal Index H	PSNR	NQI
Original	0.2097	-	-
Fractal	**0.2565**	**69.4745**	**0.9994**
Bilinear	0.2872	69.6854	0.9994
Bicubic	0.2841	67.4414	0.9993

V. CONCLUSION

In this paper, a fractional Brownian motion approach for biomedical images interpolation was developed and compared with traditional methods. The traditional methods such as bilinear and bicubic interpolation approaches, only take into account the coherence of adjoining points. Statistical self-similarity existed in Images of natural scenes was not considered. But, the fractal analysis provides the improvement for biomedical image interpolation..

The experimental results show that the fractal interpolation can obtain better image quality than that of bilinear and bicubic interpolation. Both Table 1 and Table 2 show the fractal dimension of our fBm processed tissue images that approximate original images. According to fractal index H, fractal interpolation method is better than traditional ones. Based on PSNR and NQI measures, fractal interpolated images are similar to the images obtained by bilinear and bicubic approaches. Some fractal behavior appeared in biomedical images needs to be further investigated.

According to fractal interpolation, we can easy classify breast images between normal and cancer tissue in clinical application. The breast carcinoma lesions on the tissue image were the investigated.

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