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Fuzzy Similarity Index For Discrimination Of EEG Signals

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Abstract - In this study, a new approach based on the computation of fuzzy similarity index was presented for discrimination of electroencephalogram (EEG) signals. The EEG, a highly complex signal, is one of the most common sources of information used to study brain function and neurological disorders. The analyzed EEG signals were consisted of five sets (set A – healthy volunteer, eyes open; set B – healthy volunteer, eyes closed; set C – seizure-free intervals of five patients from hippocampal formation of opposite hemisphere; set D – seizure-free intervals of five patients from epileptogenic zone; set E – epileptic seizure segments). The EEG signals were considered as chaotic signals and this consideration was tested successfully by the computation of Lyapunov exponents. The computed Lyapunov exponents were used to represent the EEG signals. The aim of the study is discriminating the EEG signals by the combination of Lyapunov exponents and fuzzy similarity index. Toward achieving this aim, fuzzy sets were obtained from the feature sets (Lyapunov exponents) of the signals under study. The results demonstrated that the similarity between the fuzzy sets of the studied signals indicated the variabilities in the EEG signals. Thus, the fuzzy similarity index could discriminate the healthy EEG segments (sets A and B) and the other three types of segments (sets C, D, and E) recorded from epileptic patients.

Key words: Fuzzy similarity index, Chaotic signal, Lyapunov exponents, Electroencephalogram (EEG) signals

I. INTRODUCTION

The electroencephalogram (EEG) is a complex and aperiodic time series which is a sum over a very large number of neuronal membrane potentials. Despite rapid advances of neuro-imaging techniques EEG recordings continue to play an important role in both, diagnosis of neurological diseases and understanding psychophysiological processes. In order to extract relevant information from recordings of brain electrical activity a variety of computerized analysis methods have been developed. Most methods are based on the assumption that the EEG is generated by a highly complex linear system, resulting in characteristic signal features like nonstationary or unpredictability [1]. Much research with nonlinear methods revealed that the EEG is generated by a chaotic neural process of low dimension [2-4]. According to these reports, the EEG has a finite noninteger correlation dimension and a positive Lyapunov exponent. Furthermore, the distinct states of brain activity had different chaotic dynamics quantified by nonlinear invariant measures such as correlation dimensions and Lyapunov exponents [2-4].

The EEG signals reflect the electrical activity of the brain. The study of the brain electrical activity, through the electroencephalographic records, is one of the most important tools for the diagnosis of neurological diseases [1,5]. The traditional analysis relies, mainly, on the detection of spectral power changes, supervised by the visual inspection of the physician: different frequency bands are considered, and the corresponding spectral powers are computed, whose changes are related to both functions and disfunctions of the central nervous system [5]. In many studies, the underlying systems generating the observed EEG signals are believed to be nonlinear or consisting of subsystems in which nonlinear mechanisms play an important role. Even when they are analyzed from healthy individuals, they manifest chaos in the nervous system [6]. Linear modeling techniques, though they allow us to deal with simplified problems, can represent the underlying system only partially, without taking into account the nonlinear contribution. Even though fairly good results have been obtained using linear modeling techniques, they seem to provide only a limited amount of information about the signal because they ignore the underlying nonlinear signal dynamics. In recent years, there has been an increasing interest in applying techniques from the domains of nonlinear analysis and chaos theory in studying the behavior of a dynamical system from an experimental time series such as EEG signals [1-4]. The purpose of these studies is to determine whether dynamical measures especially Lyapunov exponents can serve as clinically useful parameters. Estimation of the Lyapunov exponents is computationally more demanding, but estimates of these parameters are more readily interpreted with respect to the presence of chaos, as positive Lyapunov exponents are the hallmark of chaos [7].

Fuzzy theory is becoming more and more a core paradigm of research. A number of basic concepts and methods already introduced in the early stages of the theory have become standard in the application of fuzzy-theoretic tools to medical artificial intelligence subjects [8]. The notion of similarity involves an elaborate cognitive process rather than simply a mathematical model. Whenever the assessment of similarity should reproduce the judgement of a human observer based on qualitative features, it is appropriate to model it as a cognitive process that simulates human similarity perception. Among the various knowledge representation formalisms that have been proposed as ways of reasoning in the presence of uncertainty and imperfect

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knowledge, a situation typical to the human cognitive processes, fuzzy logic has very important features because:

- Fuzzy set theory has been proved a plausible tool for modeling and mimicking cognitive processes, especially those concerning recognition aspects, and
- Fuzzy set theory is able to handle qualitative nonnumerical descriptions, approximate class memberships and possibilistic reasoning [9,10].

A new method combining Lyapunov exponents and fuzzy similarity index was presented to discriminate the EEG signals. In this study, decision making was performed in two stages: feature extraction by computing the Lyapunov exponents (4 selected Lyapunov exponents) and computing fuzzy similarity index of feature sets between the reference EEG signals and the other classes of EEG signals. The data described in reference [11] was used, which is publicly available. Discrimination of the EEG signals (five sets denoted as sets A-E) could be done by the proposed method based on fuzzy similarity index.

II. LYAPUNOV EXPONENTS

Lyapunov exponents are a quantitative measure for distinguishing among the various types of orbits based upon their sensitive dependence on the initial conditions, and are used to determine the stability of any steady-state behavior, including chaotic solutions. The reason why chaotic systems show aperiodic dynamics is that phase space trajectories that have nearly identical initial states will separate from each other at an exponentially increasing rate captured by the so-called Lyapunov exponent [12,13]. This is defined as follows. Consider two (usually the nearest) neighboring points in phase space at time 0 and at time t, distances of the points in the i-th direction being $\|\delta x_i(0)\|$ and $\|\delta x_i(t)\|$, respectively. The Lyapunov exponent is then defined by the average growth rate λ_i of the initial distance,

$$\frac{\|\delta x_i(t)\|}{\|\delta x_i(0)\|} = 2^{\lambda_i t}\,(t \to \infty) \quad \text{or}$$

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} \log_2 \frac{\|\delta x_i(t)\|}{\|\delta x_i(0)\|} \qquad (1)$$

The existence of a positive Lyapunov exponent indicates chaos [12,13]. This shows that any neighboring points with infinitesimal differences at the initial state abruptly separate from each other in the i-th direction. In other words, even if the initial states are close, the final states are much different. This phenomenon is sometimes called sensitive dependence on initial conditions. Numerous methods for calculating the Lyapunov exponents have been developed during the past decade [12,13]. Generally, the Lyapunov exponents can be estimated either from the equations of motion of the dynamic system (if it is known), or from the observed time series. The latter is what is of interest due to its direct relation to the work in this paper. The idea is based on the well-known technique of state space reconstruction with delay coordinates to build a system with Lyapunov exponents identical to that of the original system from which the measurements have been observed. Generally, Lyapunov exponents can be extracted from observed signals in two different ways. The first is based on the idea of following the time-evolution of nearby points in the state space. This method provides an estimation of the largest Lyapunov exponent only. The second method is based on the estimation of local Jacobi matrices and is capable of estimating all the Lyapunov exponents. Vectors of all the Lyapunov exponents for particular systems are often called their Lyapunov spectra [12-15].

III. FUZZY SIMILARITY INDEX

Two similarity measures were proposed in the literature: one for the similarity between fuzzy sets and the other between elements in fuzzy sets [16]. The results of the studies existing in the literature have shown that the proposed measures are useful to analyze the behavior of the groups [8,10]. Similarity between fuzzy sets was studied and therefore the similarity measure between fuzzy sets were defined. To identify the change state of a system, one of the simplest methods is to compare the feature sets of the present state and ones of the previous states. If the both states are very similar, then it means that the feature sets does not show a large change, and vice versa. After the feature extraction process, a fuzzy membership function can be used to transfer the present and previous features as two fuzzy sets. The parameters of the fuzzy membership function can be determined by the features. Fuzzy sets can be obtained from the feature sets of the signals under study by repeating the fuzziness process. Suppose two fuzzy sets A and B and each set includes N features $x_1, x_2, \ldots, x_N$, a reliable and simple method presented by Lee-Kwang et al. [16] can be used to compute the similarity between the two fuzzy sets, A and B as follows:

$$S(A,B) = \frac{\sum_{i=1}^{N} (1 - |\mu_A(x_i) - \mu_B(x_i)|}{N}\,, \qquad (2)$$

where $1-|\mu_A(x_i)-\mu_B(x_i)|$ can be regarded as the similarity degree of fuzzy sets A and B on the features x_i. $S(A,B)$ is the average of the similarity degree of fuzzy sets A and B, called fuzzy similarity index. The range of $S(A,B)$ is from 0 to 1, which corresponds to the different similarity degree. $S(A,B)=1$ means the two signals are identical, otherwise there exist a difference between the two signals.

IV. RESULTS AND DISCUSSION

A. Feature Extraction by Computing Lyapunov Exponents

Feature extraction is the determination of a feature or a feature vector from a pattern vector. For pattern processing problems to be tractable requires the conversion of patterns to features, which are condensed representations of patterns, ideally containing only salient information. Selection of the inputs of pattern classification methods has two meanings: 1) which components of a pattern, or 2) which set of inputs best represent a given pattern. Dynamical measures especially Lyapunov exponents can serve as clinically useful parameters and contain a significant amount of information about the signal.

In this study, the 100 time series of 4096 samples for each class (sets A, B, C, D, and E) windowed by a rectangular window composed of 256 discrete data and then 8000 vectors (1600 vectors from each class) of 4 dimensions (dimension of the extracted feature vectors) were obtained. A rectangular window, which was formed by 256 discrete data, was selected so that the EEG signal considered to be stationary in that interval. After the required computations, 128 Lyapunov exponents were obtained for each EEG segment. The Lyapunov exponents were computed using the MATLAB software package.

Feature selection is an important component of pattern classification methods since even the best classifier will perform poorly if the features are not selected well. High-dimension of feature vectors increased computational complexity and therefore, in order to reduce the dimensionality of the extracted feature vectors (feature selection), statistics over the set of the Lyapunov exponents were used. The following statistical features were used in reducing the dimensionality of the extracted feature vectors representing the EEG signals:

1. Maximum of the Lyapunov exponents in each segment.
2. Minimum of the Lyapunov exponents in each segment.
3. Mean of the Lyapunov exponents in each segment.
4. Standard deviation of the Lyapunov exponents in each segment.

Table I presents the extracted features of five exemplary records from five classes.

B. Analysis Based on Fuzzy Similarity Index

Gaussian membership function was used to transfer present and previous features as fuzzy sets. As shown in Fig. 1, Gaussian membership functions can be used to describe 4 Lyapunov exponents. The Gaussian function depends on two parameters σ and c, and defined as follows:

$$\mu(x;\sigma,c) = \exp(-(x-c)^2/(2\sigma^2)) \qquad (3)$$

The two parameters σ and c were determined by each Lyapunov exponents. In order to describe the fuzzy characteristic of each feature, five membership functions were applied (Fig. 1). By means of, fuzzy sets were obtained from the feature sets of the signals under study. The similarity between the fuzzy sets of the signals under study indicated the variabilities in the EEG signals. The selected 4 Lyapunov exponents were used to represent the EEG signals and then obtained a fuzzy sets (5×4 matrix) by using the fuzziness process. When a segment from set A was taken as a reference segment, the fuzzy similarity index among sets A, B, C, D, E were 0.985, 0.763, 0.405, 0.392, 0.206, respectively. As expected, the first value indicating the similarity between the segments from set A is close to 1. The second value is also close to 1, since sets A and B were taken from surface EEG recordings of five healthy volunteers with eyes open and closed, respectively. Therefore, set A are most often confused with set B. The third, fourth and fifth values are close to 0 and these values indicate the significant differences between set A and the other EEG sets (segments recorded from the epileptic patients) analyzed in the classes. These results demonstrated that the fuzzy similarity index could identify the healthy EEG segments (sets A and B) and the other three types of segments (sets C, D, and E) recorded from epileptic patients. When the similarity index between healthy EEG segments and the other three epileptic segments reaches a critical level (fuzzy similarity index greater than 0.7), the risk of epilepsy should be considered. The experiments showed that the fuzzy similarity index greater than 0.7 can be considered as critical level.

TABLE I
THE EXTRACTED FEATURES OF FİVE EXEMPLARY RECORDS FROM FİVE CLASSES

Dataset	Extracted Features	Lyapunov Exponents
Set A	Maximum	0.3409
	Minimum	0.0421
	Mean	0.2074
	Standard deviation	0.0319
Set B	Maximum	0.2355
	Minimum	-0.3618
	Mean	0.1613
	Standard deviation	0.0661
Set C	Maximum	0.5716
	Minimum	-0.0387
	Mean	0.1890
	Standard deviation	0.0478
Set D	Maximum	0.7523
	Minimum	-0.1953
	Mean	0.1605
	Standard deviation	0.0745
Set E	Maximum	0.2258
	Minimum	-2.1884
	Mean	0.1578
	Standard deviation	0.2318

V. CONCLUSION

Similarity between fuzzy sets was studied and to identify the variabilities of the EEG signals the feature sets (Lyapunov exponents) of the present state and ones of the previous states were compared. The notion of similarity plays an important role in medical research and practice. Fuzzy sets were obtained from the feature sets of the EEG signals by repeating the fuzziness process. The EEG recordings have an important role in both, diagnosis of neurological diseases and understanding psychophysiological processes. Therefore, determining variabilites of the EEG signals is important in diagnosis of the neurological diseases. In order to describe the fuzzy characteristic of each feature, five Gaussian membership functions were applied. The similarity between the fuzzy sets of the signals under study were computed. The results of the present applications showed that the proposed method can be used in determining the EEG signals recorded from the epileptic patients.

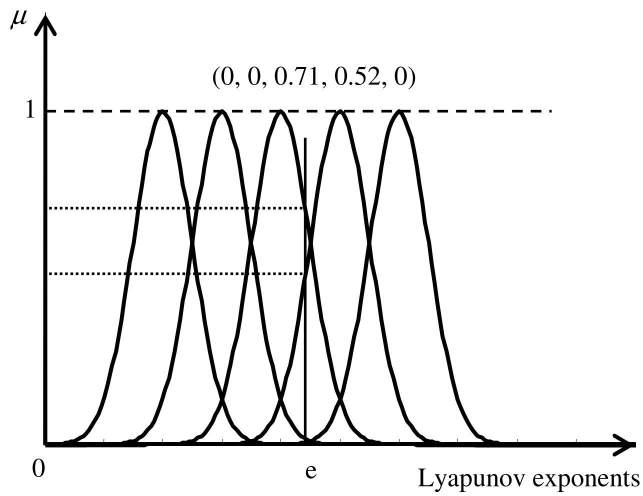

Fig. 1. Five Gaussian membership functions (μ) of the Lyapunov exponents. The feature value e is represented as a fuzzy sets of (0, 0, 0.71, 0.52, 0).

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