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Tissue Conductivity Estimation in Two-Dimension Head Model Based on Support Vector Machine

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***Abstract*—Estimating head tissue conductivity for each layer is a high dimensional, non-linear and ill-posed problem which is part of Electrical Impedance Tomography (EIT) inverse problem. Traditional methods have many difficulties in resolving this problem. Support Vector Machine (SVM) based on Statistical Learning Theory (SLT) is a new kind of learning method including Support Vector Classification (SVC) and Support Vector Regression (SVR). A new method using SVR is proposed to solve the problem in multi-input and multi-output system named Multi-SVM (MSVM). Tissue conductivity for each layer in 2-D head model is estimated effectively by MSVM. Compared with wavelet neural network method, MSVM not only obtains higher accuracy of learning, it also has greater generalization ability and faster computing speed as our experiment demonstrates.**

I. Introduction

ESTIMATING head tissue conductivity for each layer is a high dimensional, non-linear and ill-posed problem which is part of EIT inverse problem. In order to estimate head tissue conductivity, the voltages are measured on the surface. Some non-learning methods were proposed to solve this problem such as multi-start downhill simplex algorithm [1] and a finite element method (FEM) [2]. These methods not only had lower accuracy but also cost so much time that they can not meet the real-time need. Machine learning is off-line learning model. After the training process is finished, i.e., learning function is obtained, machine learning can get result very fast after test sample is fed into the learning function.

As a kind of machine learning method, Neural Network (NN) has good performances in many applications. But it is not a good learning machine since it cannot control generalization ability well following the Empirical Risk Minimization (ERM) principle. Using Wavelet Neural Network (WNN) to estimate head tissue conductivity[3], lower learning accuracy and generalization ability are presented, therefore, a more effective method is needed.

SLT based on the solid theoretical foundation, provides a new framework for the general learning problem of small-sample size statistics [4]. SVM based on SLT, can overcome the problems NN encounters. SVM includes SVC and SVR. In this paper, a new method using SVR is proposed to solve the problems in multi-input and multi-output system named MSVM. Tissue conductivity for each layer in 2-D head model for EIT inverse problem is estimated effectively by MSVM.

II. Support Vector Regression

Assume a training data set $\{(x_1, y_1),...,(x_n, y_n)\} \subset R^d \times R$, n is the number of samples and target function $f(x) = \omega \bullet x + b$. The main idea of SVR is to find appropriate w and b with the smallest generalization error ε. Thus, the regression problem can be described as [4]:

$$\min \frac{1}{2}\omega^T \omega \tag{1}$$

To allow for noise, non-negative slack variables ξ_i and a penalizing term $C > 0$ are introduced. Then the former problem can be described as:

$$\min\left(\frac{1}{2}\omega^T \omega + \frac{1}{2}C\sum_{i=1}^{n}(\xi_i + \xi_i^*)\right) \tag{2}$$

$$\text{subject to } \begin{cases} y_i - \omega \bullet x_i - b \le \varepsilon + \xi_i \\ \omega \bullet x_i + b - y_i \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{cases} \tag{3}$$

Here C is a pre-specified value, ξ_i and ξ_i^* represent upper and lower constraints on the outputs of the system respectively. To solve it, we have to solve the following QP problem by introducing Lagrangian [4]:

$$\begin{aligned} L &= \frac{1}{2}\| w \|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*) \\ &- \sum_{i=1}^{l}\alpha_i(\varepsilon + \xi_i - y_i + (\omega \cdot x_i) + b) \\ &- \sum_{i=1}^{l}\alpha_i^*(\varepsilon + \xi_i^* + y_i - (\omega \cdot x_i) - b) - \sum_{i=1}^{l}(\eta_i\xi_i + \eta_i^*\xi_i^*) \end{aligned} \tag{4}$$

α_i are Lagrange multipliers. Of all coefficients α_i, only a small number of coefficients α_i are different from zero, so they are marked as α_i^0. Each α_i^0 corresponds to a data point

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and these data points are Support Vectors (SVs). Now we can define the regression function by:

$$f(x)=\sum_{i=1}^{n}(\alpha_i-\alpha_i^*)\langle x_i,x\rangle+b \tag{5}$$

For the non-linear QP problem, it can be solved by projecting the original set of variables x in a high dimensional feature space with project Φ. But the project is often unavailable. Fortunately, it is unnecessary to know the project Φ exactly if we use kernel function: $K(x,y)=\Phi(x)\bullet\Phi(y)$. The kernel function is a symmetric function and satisfies the Mercer condition. Then the SVR is defined by:

$$f(x)=\sum_{i=1}^{n}(\alpha_i-\alpha_i^*)k(x_i,x)+b \tag{6}$$

III. METHOD

In order to compute EIT problem conveniently, head is viewed as a 2-D 3-layer concentric circle model, representing scalp layer, skull layer and brain layer respectively. $\sigma_1,\sigma_2,\sigma_3$ represent tissue conductivities, φ_1 , φ_2 , φ_3 represent voltage values and r_1,r_2,r_3 represent radius of each layer respectively (see figure 1).

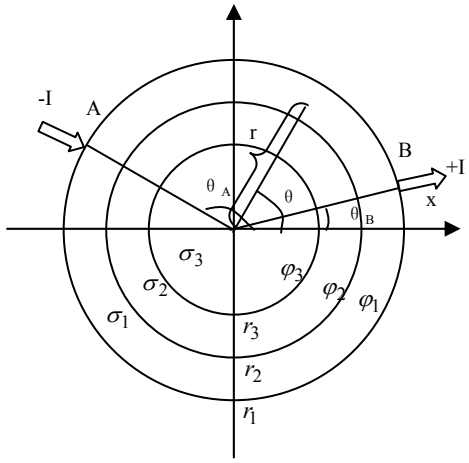

Fig. 1. 2-D 3-layer concentric circle head model

A. Forward Problems of EIT

Assume tissue conductivities of each layer are the same, analytic solution of EIT forward problem was obtained from reference [5]. Since the equations of computing voltage of 3 layers are too complicated, only the equation for computing voltage of the scalp layer is given here:

$$\varphi_1=\sum_{n=1}^{\infty}\left[g_n(1+A_n)r^n+B_nr^{-n}\right]\left[\cos n(\theta_A-\theta)-\cos n(\theta_B-\theta)\right] \tag{7}$$

$$A_n=\frac{\frac{k_{2n}}{k_{1n}}-r_2^{2n}}{r_2^{2n}-r_1^{2n}-\frac{k_{2n}}{k_{1n}}} \tag{8}$$

$$B_n=r_1^{2n}g_nA_n \tag{9}$$

$$k_{1n}=\left(1+\frac{\sigma_3}{\sigma_2}\right)\left(1+\frac{\sigma_2}{\sigma_1}\right)+\left(\frac{r_3}{r_2}\right)^{2n}\left(1-\frac{\sigma_3}{\sigma_2}\right)\left(1-\frac{\sigma_2}{\sigma_1}\right) \tag{10}$$

$$k_{2n}=\frac{2\sigma_2}{\sigma_1}\left[r_2^{2n}\left(1+\frac{\sigma_3}{\sigma_2}\right)-r_1^{2n}\left(1-\frac{\sigma_3}{\sigma_2}\right)\right] \tag{11}$$

$$g_n=\frac{1}{n\pi\sigma_1r_1^n} \tag{12}$$

Here I is intensity of infused electrical current. θ_A,θ_B,θ are angles of infused electrical current, output electrical current and viewing point respectively. r is the distance between viewing point and the centre of circle.

B. Estimating tissue conductivity

Given conductivity data of each layer, we can obtain d vectors of voltage distribution on the scalp by forward problem computation. Likewise, we can obtain multiple groups of d vectors if multiple groups of conductivity data for each layer is are provided. However, the problem is to compute tissue conductivity $\sigma_1,\sigma_2,\sigma_3$ (multi-output) using multi-group d vectors of voltage distribution (multi-input). It is a multi-input and multi-output system. Obviously, since known number of equations is more than unknown number of variables, the system of equations is over determined. Using MSVM, we can establish the relationship between inputs and outputs. Under the circumstances of unknown conductivity of each layer, we can measure the values of voltage on the edge after electrical current is infused to edge of target, and then train them with MSVM to regress $\sigma_1,\sigma_2,\sigma_3$.

C. MSVM

Voltage values measured by 16 scalp electrodes are viewed as inputs, and conductivity $\sigma_1,\sigma_2,\sigma_3$ are viewed as outputs, so it is a multi-input and multi-output system. Regression problem can be described as:

$$\text{D}=\{(x_1,y_1),\cdots\cdots,(x_n,y_n)\},\ x\in R^d,y\in R^m \tag{13}$$

In order to overcome this problem, we introduce non-negative slack variables ξ_{ij}, ξ_{ij}^* and C_j for each output. Thus, the problem can be described as follows:

$$\min\left\{\sum_{j=1}^{m}\left[\frac{1}{2}\|\omega_j\|^2+C_j\sum_{i=1}^{n}(\xi_{ij}+\xi_{ij}^*)\right]\right\} \tag{14}$$

$$\text{subject to}\quad\begin{cases}y_{ij}-\omega_j^T\phi_j(x_{ij})-b_j\le\varepsilon_j+\xi_{ij}\\ \omega_j^T\phi_j(x_{ij})+b_j-y_{ij}\le\varepsilon_j+\xi_{ij}^*\\ \xi_{ij},\xi_{ij}^*\ge0\end{cases} \tag{15}$$

So this kind of multi-input and multi-output problem can be solved with m-independent SVR:

$$\min\left[\frac{1}{2}\|\omega_1\|^2+C_1\sum_{i=1}^{n}(\xi_{i1}+\xi_{i1}^*)\right]+\cdots$$
$$+\min\left[\frac{1}{2}\|\omega_m\|^2+C_m\sum_{i=1}^{n}(\xi_{im}+\xi_{im}^*)\right] \tag{16}$$

IV. Experimental Result

In the 3-layer concentric circle head model of 2-D, the ranges of parameters are shown as Table I.

TABLE I.
RANGES OF PARAMETERS FOR 3-LAYER CONCENTRIC CIRCLE HEAD MODEL

Tissue	Brain	Skull	Scalp
Resistivity range (Ω/m)	0.9—1.1	200—250	2.5—3.4
Radius (cm)	8.7	9.2	10

For each layer, 10 groups of resistivity are selected equably between maximum value and minimum value, so the total number of groups is 1000. These groups are randomly divided into two parts, one for training data including 950 groups, the other for test data including 50 groups (test set NO. 1). In order to avoid the relativity between training data and test data, we randomly produce other test data of 50 groups (test set NO. 2).

The locations of 16 electrodes are equably distributed on the scalp. We can obtain various voltage distributions through forward computing the 1050 groups of resistivity. In order to appraise the effect of training and test, Error Ratio for Each Layer ($EREL_i$), Maximum Error Ratio for Each Layer ($MEREL_i$), Systemic Relative Error Ratio of System ($SRERS_j$) and Total Relative Error Ratio ($TRER$) are selected as standard criteria:

$$EREL_i = \sqrt{\left(\sum_{j=1}^{n} |\tilde{y}_{ji} - y_{ji}|^2\right) / \left(\sum_{j=1}^{n} y_{ji}^2\right)} \times 100\% \quad (17)$$

$$MEREL_i = \max_{j=1}^{n} (|\tilde{y}_{ji} - y_{ji}|) / y_{ji} \times 100\%) \quad (18)$$

$$SRERS_j = \sqrt{\left(\sum_{i=1}^{m} |\tilde{y}_{ji} - y_{ji}|^2\right) / \left(\sum_{i=1}^{m} y_{ji}^2\right)} \times 100\% \quad (19)$$

$$TRER = \sqrt{\left(\sum_{j=1}^{m}\sum_{i=1}^{n} |\tilde{y}_{ji} - y_{ji}|^2\right) / \left(\sum_{j=1}^{m}\sum_{i=1}^{n} y_{ji}^2\right)} \times 100\% \quad (20)$$

Here m is the number of outputs, n is the number of training or test samples, y_{ji} is the analytic solutions, and $\tilde{y}_{ji}$ is the solutions of SVR.

In our experiment, sequential minimal optimization (SMO) algorithm is used to train all data [6][7], $\varepsilon = 0.01$, C=100. Gaussian Radial Basis Function (RBF) is selected as kernel function: for Scalp layer, $2\sigma^2$ =1; for Skull layer, $2\sigma^2$ =10; for Brain layer, $2\sigma^2$ =1. The results of trainings and tests are shown in Table II.

For test set NO. 1 and test set NO. 2, figures 2, 3, 4 show the solutions of real resistivities, the solutions of resistivities regressed by MSVM of 3 layers respectively; Figures 5, 6, 7 show test $EREL_i$; Figure 8 shows the $SRERS_j$.

TABLE II
THE RESULTS OF TRAININGS AND TESTS

	Training set	Test set NO.1	Test set NO.2
$EREL_i$ for Scalp	0.1692%	0.1683%	0.1604%
$EREL_i$ for Skull	0.2154%	0.2186%	0.1902%
$EREL_i$ for Brain	0.5584%	0.5710%	0.4973%
$MEREL_i$ for Scalp	0.3846%	0.3297%	0.3852%
$MEREL_i$ for Skull	0.8143%	0.4972%	0.4894%
$MEREL_i$ for Brain	2.254%	1.647%	0.9884%
Maximum $SRERS_j$	0.8144%	0.4972%	0.4894%
$TRER$	0.2154%	0.2186%	0.1902%

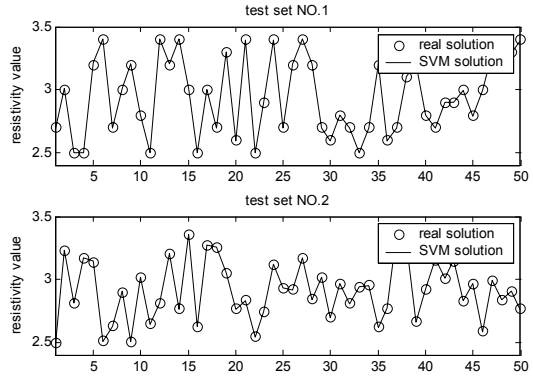

Fig. 2. The solution of real resistivity and resistivity regressed by MSVM of scalp layer

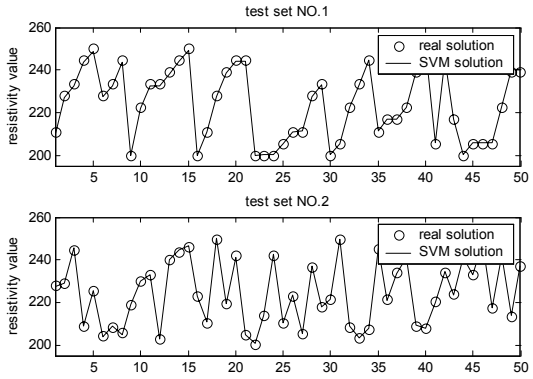

Fig. 3. The solution of real resistivity and resistivity regressed by MSVM of skull layer

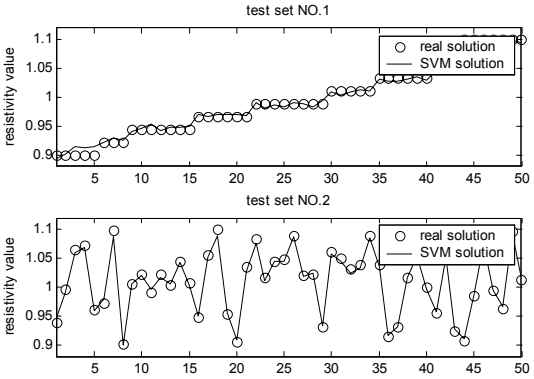

Fig. 4. The solution of real resistivity and resistivity regressed by MSVM of brain layer

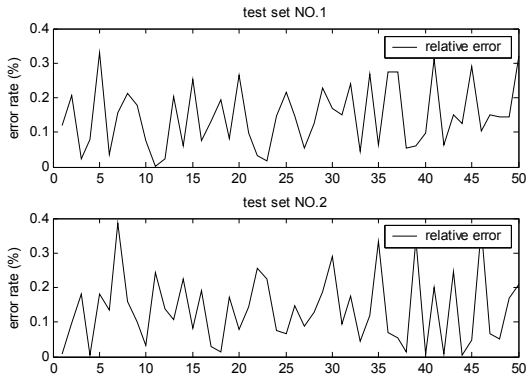

Fig. 5. Test relative error of scalp layer

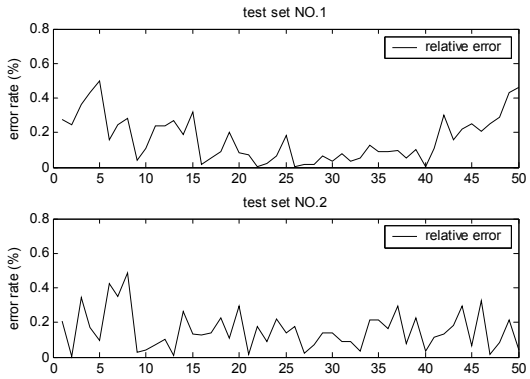

Fig. 6. Test relative error of skull layer

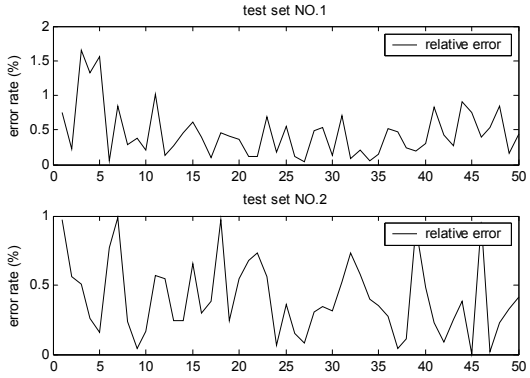

Fig. 7. Test relative error of brain layer

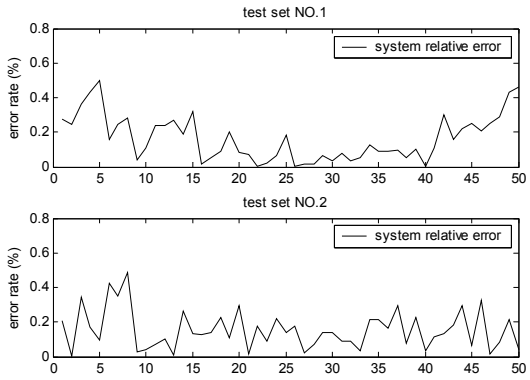

Fig. 8. $SRERS_j$

For the same problem, WNN is used to estimate tissue conductivity [8]. The performances of two methods are shown in Table III. It manifests that MSVM outperforms WNN.

TABLE III
THE PERFORMANCES OF MSVM AND WNN

	TRER of training set	*TRER* of test set NO.1	*TRER* of test set NO. 2
SVM	0.2154%	0.2186%	0.1902%
WNN	3.56%	13.09%	

V. CONCLUSION

Since the electrodes are located on the scalp layer, measured voltages on the surface are not sensitive to the changes of conductivity in the central area. Thus, $EREL_i$ and $MEREL_i$ represent the trend of gradual increase from outside to inside. As our experiment demonstrates, training effect of SVR is satisfactory: $EREL_i$ is less than 0.6%, and $MEREL_i$ is less than 2.3%. Of test set NO. 1 and test set NO. 2, test effect is also better: $EREL_i$ is less than 0.6%, and $MEREL_i$ is less than 1.7%. Compared with wavelet neural network, MSVM has higher learning accuracy and greater generalization ability.

SVM is an off-line learning model. After the process of training is finished, i.e., learning function is obtained, SVM can get result very fast after new voltages are input. So it can meet the real-time need for EIT inverse problem. Thus, MSVM exhibits its great potential to solve other multi-input and multi-output problem.

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