

# Movement Time Planning in Human Movement with Via-Points

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**Abstract**—In previous research, even though various models that reproduce movement trajectory have already been proposed, a movement time planning criterion has never been proposed. However, a possible computational model that can estimate via-point time in complex trajectories has been proposed that suggests that the via-point time average of the integration of the square of the smoothness of the motor commands between each via-point is equal. In this report, we measured three kinds of via-point reaching movements and then performed statistical tests to investigate the computational theory. Results suggest that their computational theory is valid.

## I. INTRODUCTION

Several possible computational models have been proposed for human arm movement control and planning, such as minimum jerk criterion [1], minimum commanded torque change criterion [2][3], and minimum motor command change criterion [4]. To generate trajectory, these models require the location of the start and end points, and movement time as boundary conditions. However, we hypothesize that movement time is not necessarily required as a constrained condition, but is decided depending on task required precision.

In this paper, we consider a complex trajectory that passes through several via-points. In this case, movement time can be classified into two kinds: entire duration and duration between via-points. Concerning the former, Fitts' Law [5] predicts movement time by end point accuracy. For the latter, Wada & Kawato [6] proposed a possible computational trajectory generation model that estimates the time to pass through via-points. In via-point movement, this model determines the time to pass through via-points by the optimization of movement criterion. That is, the model suggests that trajectory formation does not need priori temporal information about via-points.

In the via-point time optimization algorithm, a cost function is optimized on the condition that the via-point time average of the integration of the square of the smoothness of the motor command between each via-point is equal. Thus, the possibility is suggested that CNS plans via-point time to equalize the via-point time average of the integration of the square of the smoothness of the motor commands between each via-point.

In this paper, we measured the reaching movement with one or two via-points to investigate the possibility of the computational theory that trajectory is generated to equalize

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## II. VIA-POINT TIME OPTIMIZATION MODEL

In trajectory formation passing through via-points, a via-point time optimization model [6] decides the via-point time to minimize the commanded torque change, as shown in the following equation:

$$C_\tau = \int_0^{t_f} \sum_{k=1}^K \left( \frac{d\tau^k}{dt} \right)^2 dt$$

$$= \sum_i C_i(t_i) = \sum_i \int_{t_{i-1}}^{t_i} \sum_{k=1}^K \left( \frac{d\tau_i^k}{dt} \right)^2 dt, \quad (1)$$

where  $t_f$  is the entire motion duration,  $i$  is the indices of via-points ( $i = 1, 2, \dots, n$ ),  $t_i$  is the time passing through the via-point  $i$ ,  $\tau^k$  is the commanded torque of joint  $k$ , and  $K$  is the number of joints.

This model estimates via-point time as follows.

Step 1 A trajectory is generated by Forward Inverse Relaxation Model (FIRM) [7] according to a set of initial via-point times.

Step 2 The movement time between each via-point is updated to reduce the performance index of the minimum commanded torque change using the steepest descent method. Then the entire movement time is basically lengthened.

Step 3 To satisfy the given entire motion duration, the via-point time obtained in Step 2 is corrected.

In Step 2, the via-point time  $t_i$  must be extended by the steepest descent method. The movement time between each via-point is updated with the following equation:

$$\Delta t_i = \varepsilon \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \sum_{k=1}^K \left( \frac{d\tau_i^{*k}}{dt} \right)^2 dt, \quad (2)$$

where  $\tau^*$  represents the commanded torque by which  $C_i$  is minimized and  $\varepsilon$  is an appropriate positive coefficient.

The via-point time is corrected according to the following equation:

$$t_i \leftarrow \frac{t_i + \Delta t_i}{t_f + \Delta t_f} t_f, \quad (3)$$

where  $\Delta t_f = \sum \Delta t_i$ . Thus, the given entire motion duration is satisfied.

In the above optimization algorithm, if (4), which is the via-point time average of the integration of the square of

the smoothness of the commanded torque between each via-point, is equal, cost function (1) converges and each via-point time ( $t_1, t_2, \dots, t_n$ ) is optimized [6].

$$C_i^{via} = \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \sum_{k=1}^K \left( \frac{d\tau_i^k}{dt} \right)^2 dt \quad (4)$$

Thus, by assuming that a human generates trajectory according to the minimum commanded torque change criterion, in trajectory formation passing through via-points, it is suggested that trajectory is generated to equalize (4) between each via-point. To validate the possibility that trajectory is generated to equalize (5), which is the double-joint version of (4) for each via-point interval, we performed experiments using human movements with via-points.

$$C_i^{via} = \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \left( \left( \frac{d\tau_i^{sld}}{dt} \right)^2 + \left( \frac{d\tau_i^{elb}}{dt} \right)^2 \right) dt, \quad (5)$$

Here,  $\tau^{sld}$  is shoulder torque and  $\tau^{elb}$  is elbow torque.

### III. METHOD

#### A. Procedure and Task

In each experimental task, the subjects were seven right-handed males, 21-22 years old. The experimental setup is shown in Fig. 1. Subjects sat in chairs adjusted to lift their arms to shoulder level, and their wrists were supported by a brace mounted on an air-sled to reduce the influence of friction. The positions of the shoulder, elbow, and hand were measured by OPTOTRAK3020, which detected infrared makers at a sampling frequency of 500 (Hz). Subjects were given online feedback of hand position as a cursor indicated on the monitor.

In this experiment, two reaching tasks with one via-point (Fig. 2: A; Task 1, B; Task 2) and a reaching task with two via-points (Fig. 2: C; Task 3) were designed. Subjects moved their arms from an initial to a final position by passing

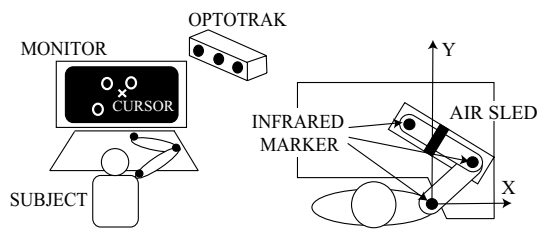


Fig. 1. Experimental setup

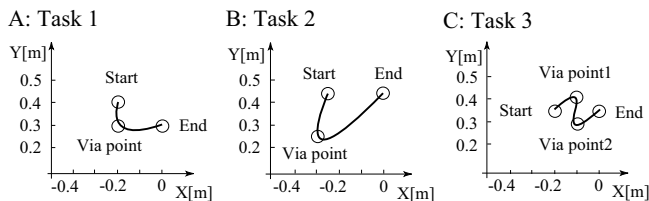


Fig. 2. Start point, via point and end point in experimental tasks

through via-points shown on the monitor as a circle with a 2 (cm) radius. Subjects were instructed to move within a time limit (Task 1: 0.5 (s), Task 2: 1 (s), and Task 3: 1 (s)); time constraints are rough indications or guides rather than rigid constraints. In each task, subjects performed 30-40 trials after many practices.

#### B. Data Analysis

To calculate the value of  $C_i^{via}$ , the via-point locations (via-point time) in the measured trajectories must be estimated. A via-point estimation algorithm was proposed by Wada & Kawato [9], which we used in this paper to estimate via-point time. Furthermore, we checked via-point positions by another method: they were estimated as locations corresponding to the locations of the local minimum of tangential velocity.

Position data were filtered by a third order Butter-worth filter with a cutoff frequency of 15 (Hz). The actual starting and ending positions of each movement were determined using tangential velocity with 5% of maximum threshold. The commanded torque ( $\tau_1, \tau_2$ ) was computed from the following inverse dynamics model (IDM):

$$\begin{aligned} \tau_1 &= (I_1 + I_2 + 2M_2L_1S_2 \cos(\theta_2) + M_2(L_1)^2)\ddot{\theta}_1 \\ &\quad + (I_2 + M_2L_1S_2 \cos(\theta_2))\ddot{\theta}_2 \\ &\quad - (M_2L_1S_2(2\dot{\theta}_1 + \dot{\theta}_2))\dot{\theta}_2 \sin(\theta_2) \\ &\quad + B_{11}\dot{\theta}_1 + B_{12}\dot{\theta}_2 \\ \tau_2 &= (I_2 + M_2I_1S_2 \cos(\theta_2))\ddot{\theta}_1 + I_2\ddot{\theta}_2 \\ &\quad + M_2L_1S_2(\dot{\theta}_1)^2 \sin(\theta_2) + B_{21}\dot{\theta}_1 + B_{22}\dot{\theta}_2. \end{aligned}$$

Here,  $\theta_i$ ,  $\dot{\theta}_i$ , and  $\ddot{\theta}_i$  represent the position, velocity, and acceleration of each joint, respectively.  $M_i, L_i, S_i, I_i, B_{ij}$  represent the mass, length, distance from the mass center to the joint, the rotary inertia of link  $i$  around the joint, and the coefficients of viscosity, respectively.  $B_{ij}$  shows that the joint angle velocity of link  $j$  influences the actuated torque of link  $i$ . Joints 1 and 2 correspond to the shoulder and elbow, respectively. The parameters of dynamics were estimated by the methods of Gomi & Osu [8] and Nakano et al. [3]. The ranges of the parameters of dynamics of all subjects are shown in Table I.

The following trajectories were rejected as failures; 78% trials of all experiments were accepted.

- 1) Trajectories that include some correction movements.
- 2) Trajectories whose local minimum velocity was less than 5% of maximum velocity.

TABLE I

THE RANGES OF PARAMETERS OF DYNAMICS OF ALL SUBJECTS

Parameter	Link1	Link2
$L_i$ (m)	0.21-0.28	0.31-0.35
$M_i$ (kg)	0.98-1.40	1.01-1.13
$S_i$ (m)	0.074-0.106	0.153-0.171
$I_i$ (kg m <sup>2</sup> )	0.0083-0.0242	0.0351-0.0495
$B_i$ (kg m <sup>2</sup> /s)	0.65-0.76	0.78-0.92
$B_{ij}$ (kg m <sup>2</sup> /s)	0.18-0.21	0.18-0.21

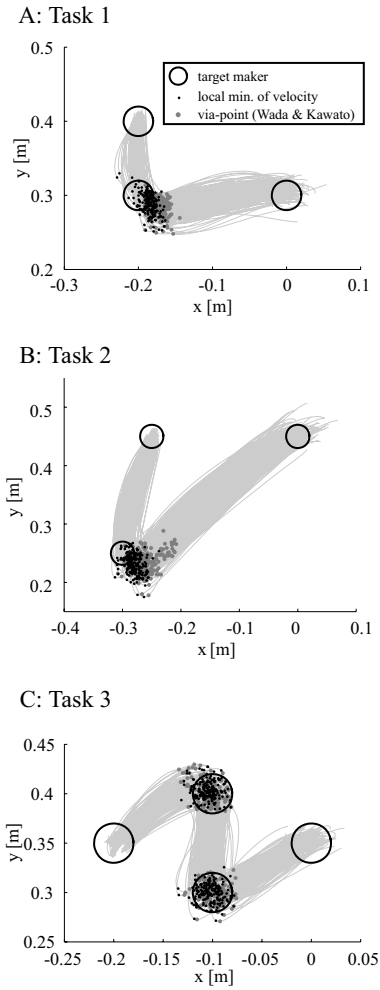


Fig. 3. Via-points estimated by Wada & Kawato's method and local minima of velocity.

- 3) Trajectories whose local minimum did not exist in the velocity profiles.

## IV. RESULTS

### A. Estimated Via-Points

The via-points estimated from the measured trajectories by the two methods are shown in Fig. 3. The via-points estimated by Wada & Kawato's method are approximated slightly later than the local minimum of the tangential velocity profile, which is one feature of minimum commanded torque change trajectories [2].

### B. Statistical Results

1) *Single Regression Analysis*: First, we show the results of simple linear regression analysis:  $C_i^{via} = \alpha \cdot C_j^{via}$  in Table II. In one via-point task (Task 1 and 2),  $C_1^{via}$  is the via-point time average of the commanded torque change from the start to the via-point,  $C_2^{via}$  is the value from the via-point to the end point, in two via-points task (Task 3),  $C_1^{via}$  is the value from the start point to via-point 1,  $C_2^{via}$  is the value from

via-point 1 to via-point 2 and  $C_3^{via}$  is the value from via-point 2 to the end point. In the analysis of Task 3  $C_1^{via}-C_2^{via}$ ,  $C_2^{via}-C_3^{via}$ , and  $C_1^{via}-C_3^{via}$  were unified.

The determination coefficients for all subjects are significant ( $p < .0001$ ), and the slopes of the regression lines of most subjects are approximately 1 (see Table II). The analysis results using all subject data are shown in Table II as "all". In the results for any tasks, the slope of the regression line using all subject data is also approximately 1. Thus, the results of simple linear regression analysis suggest that trajectory is generated to equalize the via-point time average of the commanded torque change between each via-point.

2) *Confidence Interval of Population Rate*: Next, we show the results of the estimation of the 95% confidence interval of population rate  $C_j^{via}/C_i^{via}$  in Table III. In one via-point task (Task 1 and 2), the population rate was estimated from  $C_2^{via}/C_1^{via}$ . In two via-points task (Task 3), the population rate was estimated from unified data  $C_2^{via}/C_1^{via}$ ,  $C_3^{via}/C_2^{via}$ , and  $C_3^{via}/C_1^{via}$  as well as the above simple linear regression analysis.

The population rates of the  $C_j^{via}/C_i^{via}$  of most subjects are distributed in approximately 1. For all subjects ("all" in Table III), the population rate of  $C_j^{via}/C_i^{via}$  is also distributed in approximately 1 in the results for all tasks. This tendency means that each value of  $C_i^{via}$  is almost equal. Thus, the estimation results of the population rate also suggest that trajectory is generated to equalize the via-point time average of the commanded torque change between via-points.

### C. Distribution of entire movement time

The distributions of the entire movement time of all evaluated trajectories are shown in Fig. 4. In all tasks, the distributions of the entire movement time vary widely (Task 1:  $0.82 \pm 0.19$ (s), Task 2:  $1.05 \pm 0.23$  (s), Task 3:  $1.16 \pm 0.22$  (s)). This means that the observed tendency can be observed in various movement times.

## V. CONCLUSION

In this study, we measured three kinds of via-point reaching movements performed by seven subjects to investigate the validity of a computational theory that argues that trajectory is generated to equalize the via-point time average of the commanded torque change between via-points. Results suggest a tendency that supports the above computational theory.

The double joint commanded torque evaluated in this report was approximately estimated by dynamic equations.

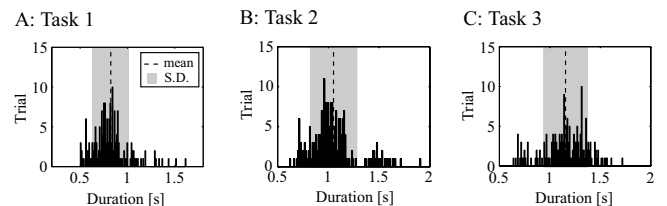


Fig. 4. Distribution of entire movement time of all evaluated trajectories

TABLE II  
RESULTS OF SIMPLE LINEAR REGRESSION ANALYSIS

Subject	Method 1: Using via-point estimation method (Wada & Kawato)					
	Task 1		Task 2		Task 3	
	$R_2$	$\alpha$ (t value, df)	$R^2$	$\alpha$ (t value, df)	$R^2$	$\alpha$ (t value, df)
TA	0.949	0.975*** (21.17, 24)	0.934	0.483*** (20.47, 29)	0.455	0.558*** (9.31, 104)
KT	0.881	1.085*** (12.48, 21)	0.879	0.987*** (14.27, 28)	0.691	0.744*** (11.50, 59)
AI	0.927	1.087*** (19.25, 29)	0.855	1.075*** (10.59, 19)	0.867	0.810*** (16.38, 41)
KK	0.907	1.281*** (17.93, 33)	0.744	0.642*** (9.95, 34)	0.770	0.915*** (18.67, 104)
AS	0.835	0.729*** (11.24, 25)	0.814	0.587*** (12.02, 33)	0.660	0.602*** (13.99, 101)
TS	0.827	0.808*** (10.25, 22)	0.779	0.846*** (9.93, 28)	0.786	0.837*** (15.81, 68)
MT	0.882	1.026*** (9.87, 13)	0.924	1.063*** (17.78, 26)	0.709	0.849*** (10.36, 44)
all	0.901	0.972*** (39.78, 173)	0.831	0.941*** (31.57, 203)	0.843	0.800*** (53.11, 527)
	Method 2: Using local minima of velocity					
	Task 1		Task 2		Task 3	
	$R_2$	$\alpha$ (t value, df)	$R^2$	$\alpha$ (t value, df)	$R^2$	$\alpha$ (t value, df)
TA	0.946	0.944*** (20.60, 24)	0.930	0.507*** (19.63, 29)	0.467	0.578*** (9.54, 104)
KT	0.809	1.245*** (9.42, 21)	0.840	1.020*** (12.12, 28)	0.645	0.697*** (10.35, 59)
AI	0.917	1.181*** (17.95, 29)	0.848	1.103*** (10.29, 19)	0.875	0.817*** (16.93, 41)
KK	0.773	1.324*** (10.62, 33)	0.699	0.685*** (8.89, 34)	0.782	0.961*** (19.29, 104)
AS	0.718	0.736*** (7.983, 25)	0.800	0.558*** (11.47, 33)	0.716	0.587*** (15.95, 101)
TS	0.906	0.691*** (14.54, 22)	0.722	0.887*** (8.53, 28)	0.795	0.865*** (16.25, 68)
MT	0.918	0.972*** (12.05, 13)	0.899	1.064*** (15.25, 26)	0.665	0.873*** (9.34, 44)
all	0.852	0.962*** (31.53, 173)	0.806	0.968*** (29.07, 203)	0.847	0.803*** (53.95, 527)

\* $p < .05$ , \*\* $p < .01$ , \*\*\* $p < .0001$

TABLE III

95% CONFIDENCE INTERVAL OF POPULATION RATE OF  $C_j^{via}/C_i^{via}$  (METHOD 1: USING VIA-POINT ESTIMATION METHOD (WADA & KAWATO), METHOD 2: USING LOCAL MINIMA OF VELOCITY)

Subject	95% C.I. of population rate of $C_j^{via}/C_i^{via}$					
	Method 1			Method 2		
	Task 1	Task 2	Task 3	Task 1	Task 2	Task 3
TA	0.907-1.122	0.450-0.556	0.729-1.063	0.870-1.084	0.485-0.593	0.787-1.140
KT	0.945-1.337	0.837-1.144	0.810-1.198	1.074-1.905	0.876-1.296	0.823-1.308
AI	0.962-1.202	0.965-1.376	0.784-1.068	1.044-1.351	0.998-1.451	0.793-1.068
KK	1.123-1.441	0.729-0.973	1.112-1.424	1.164-1.834	0.813-1.120	1.174-1.503
AS	0.624-0.875	0.573-0.838	0.647-0.843	0.628-1.007	0.567-0.855	0.628-0.795
TS	0.717-1.055	0.700-1.007	0.908-1.244	0.602-0.803	0.757-1.170	0.953-1.227
MT	0.906-1.568	0.956-1.224	1.023-1.437	0.859-1.401	0.999-1.326	1.147-1.670
all	0.992-1.119	0.802-0.915	0.945-1.070	1.032-1.235	0.861-0.997	0.986-1.120

In future studies, the validity of the above computational theory should be investigated at higher levels such as the electromyogram (EMG) level. However, such research has to wait for appropriate models that estimate dynamic torque from EMG with high accuracy. In drawing and writing movements that are more complex than via-point reaching movements, a phenomenon known as isochrony has been reported whose movement duration is only weakly dependent on movement extent [5][10]. Perhaps isochrony can be deduced from the computational theory of a via-point time optimization model. We will carry out more detailed and broader studies to approach the movement time planning in human arm movements.

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