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Speckle Reduction in Medical Ultrasound: A Novel Scatterer Density Weighted Nonlinear Diffusion Algorithm Implemented as a Neural-Network Filter

Ahmed M. Badawi, *Senior Member, IEEE*, and Muhammad A. Rushdi

***Abstract*—. This paper proposes a novel algorithm for speckle reduction in medical ultrasound imaging while preserving the edges with the added advantages of adaptive noise filtering and speed. We propose a nonlinear image diffusion algorithm that incorporates two local parameters of image quality, namely, scatterer density and texture-based contrast in addition to gradient, to weight the nonlinear diffusion process. The scatterer density is proposed to replace the existing traditional measures of quality of the ultrasound diffusion process such as MSE, RMSE, SNR, and PSNR. This novel diffusion filter was then implemented using backpropagation neural network for fast parallel processing of volumetric images. The experimental results show that weighting the image diffusion with these parameters produces better noise reduction and produces a better edge detection quality with reasonable computational cost. The proposed filter can be used as a preprocessing phase before applying any ultrasound segmentation or active contour model processes.**

I. INTRODUCTION

Medical ultrasound is a mode of medical imaging that has a wide array of clinical applications, both as a primary modality and as an adjunct to other diagnostic procedures [1]. The clinical utility of ultrasound imaging is in large part due to three characteristics. These are that ultrasound is a real-time modality, does not utilize ionizing radiation, and provides quantitative measurement and imaging of blood flow. However, an inherent characteristic of ultrasound imaging and any type of coherent imaging in general, is the presence of speckle noise. Speckle is a random interference pattern in an image formed with coherent radiation of a medium containing many sub-resolution scatterers. The texture of the observed speckle pattern does not correspond to underlying structure. Speckle has a negative impact on ultrasound imaging. Bamber and Daft show a reduction of lesion detectability of approximately a factor of eight due to the presence of speckle in the image [2]. This radical reduction in contrast resolution is responsible for the poorer effective resolution of ultrasound compared to X-ray and MRI.

Therefore, the methods of speckle reduction have been established in the past 40 years as one of the active research fields in medical ultrasound information processing [3].

A. M. Badawi is with the Department of Mechanical, Aerospace, and Biomedical Engineering, University of Tennessee, Knoxville, TN 37996 USA (phone: 865-974-6009; fax: 865-946-1787; e-mail: ambadawi@utk.edu). A. M. Badawi is an associate professor of Systems & Biomedical Engineering, Cairo University, Egypt.
M. A. Rushdi, is with the Department of Electrical & Computer Engineering, University of Miami, Coral Gables, FL 33146 USA (e-mail: m.rushdi@umiami.edu)

Adaptive filtering for speckle reduction has been studied by [7] in order to reduce speckles in ultrasound images. However, denoising techniques should not only reduce the noise, but do so without blurring or changing the location of the edges. Hence, new techniques, based on the use of partial differential equations, have been extensively studied since the early work of Perona and Malik in 1987 [8] and others [9-16]. The idea behind the use of the diffusion equation in image processing arose from the use of the Gaussian filter in multi-scale image analysis where an image is convolved with a Gaussian filter. If the diffusivity function is a constant, i.e., independent of image positions (x, y) or time (t), it leads to a linear diffusion equation [8], with a homogeneous diffusivity. In this case, all locations in the image, including the edges are smoothed equally. This is, of course, undesirable, and a simple improvement would be to change diffusivity with the location x and y in the image, thus converting the equation into a linear diffusion equation with non-homogeneous diffusivity. If the diffusivity function is image dependent, then the linear diffusion equation becomes a non-linear diffusion equation [10-16]. For example, by using a function that was based on the derivative of the image at time t, Perona and Malik [9] were able to control the diffusion near the edges in the image. Since the diffusivity is a scalar, terminology from partial differential equations refer this case as isotropic non-linear diffusion. Anisotropic diffusion is the case where the diffusivity function is a tensor-valued function, varying with both the edge location and its orientation [10-16]. Thus, diffusion across the edge can be prevented while allowing diffusion along the edge. This prevents the edge from being smoothed during the denoising process. There are several factors that must be considered in the use of diffusion-based techniques for denoising. These include the choice of the diffusivity function, setting any parameters used in the diffusivity function [11-14], the method of discretization of the PDE, the options used in the solution of the PDE such as the time for which it is evolved, the method used for solving the linear system of equations, etc. Mean square error and signal to noise ratio were used for evaluating denoising process [17-18]. Neural networks for image processing were investigated in the literature by several authors in order to implement a certain nonlinear function or transform [20-24]. Signal-to-noise ratio (SNR) and Peak signal-to-noise-ratio image measure (PSNR) derived from the root mean squared error (RMSE) as image quality measures in compression, representation, and standards were described in details in [29]. Higher quality measures do not always mean better visual quality of enhanced edges and denoised structures.

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Our paper proposes novel ultrasound image geometry, quality and noise parameters that can define ultrasound image quality measures better than the traditional ones and are used to weight a nonlinear diffusion filter for speckle reduction. The resultant diffusion filter combines the advantages of locally adaptive filtering and computational speed. The rest of this paper is organized as follows. Section 2.1 gives a brief introduction to the nonlinear diffusion filtering scheme and its applicability in image denoising. Then, in Section 2.2, two image quality and noisiness measures are introduced to produce a weighted diffusion scheme that performs better than existing ones. The experimental analysis and results are summarized in Section 3. Finally, Section 4 gives conclusions and suggestions for future work to speed up the diffusion process.

II. Weighted Nonlinear Diffusion In ultrasound Imaging

A. Nonlinear diffusion in image denoising

Diffusion is intuitively regarded as a physical process that equilibrates concentration differences without creating or destroying mass. This physical observation can be easily cast in a mathematical formulation. If u is the concentration and D is the diffusion tensor, then the diffusion equation is given as:

$$\partial_t u = div(D\nabla u)\,. \qquad (1)$$

In image processing we may identify the concentration u with the gray value at a certain location. If the diffusion tensor D is constant over the whole image domain, one speaks of *homogeneous* diffusion, and a space-dependent filtering is called *inhomogeneous*. Often the diffusion tensor is a function of the differential structure of the evolving image itself. Such a feedback leads to *nonlinear diffusion* filters.

The diffusion-based filter calculates a filtered image $u(x,y,t)$ of the original noisy image $f(x,y)$ as a solution to the nonlinear diffusion equation as shown:

$$\partial_t u = div(D(x,y)\nabla u) \qquad (2)$$

with the original image $f(x,y)$ as the initial state:

$$u(x,y,0)=f(x,y), \qquad (3)$$

and reflecting boundary conditions on the image boundary:

$$\partial_n u := 0\,, \qquad (4)$$

where n denotes the normal to the image boundary. The nonlinear diffusivity function $D(x,y)$ is usually given as a strictly decreasing function of the magnitude of the gradient.

B. Image quality measures for weighting nonlinear diffusion

Perona and Malik [9] suggested incorporating the image gradient into image diffusion-based filtering scheme to produce adaptive edge-preserving image filters. Since then, many researchers [10-16] have suggested improvements and modifications related to the form of the diffusivity function and the terms of the diffusion PDE [17-18]. However, diffusion schemes that depend on other factors of physical importance haven't received much attention in literature. Here, we investigate the effect of three important image features on the performance of ultrasound image diffusion scheme. The features, namely the image gradient, the texture-based contrast, and the ultrasound scatterer density parameter, describe the geometric, resolution, and noise properties, respectively.

B.1. Image Gradient

Gradient edge detection is the most widely used technique to weight the nonlinear diffusion filters. Here, the image is convolved with only two kernels, one estimating the gradient in the x-direction, Gx, the other is the gradient in the y-direction, Gy. The absolute gradient magnitude is then given by:

$$|G| = \sqrt{G_x^2 + G_y^2}\ , \qquad (5)$$

and is often approximated with

$$|G| = |G_x| + |G_y|\,. \qquad (6)$$

B.2. Texture-Based Contrast

Characteristics of the second order describe interactions of neighbors in the texture. Large set of characteristics is derived from the so-called gray level cooccurence matrix. Cooccurrence matrices, originally called gray-tone spatial dependency matrices, were introduced by Haralick et al. [4], who used them to define textural properties of images. These texture features has been used by the first author to construct a part of the feature vector used in diffused liver tissue characterization classification problem [25-28]. These second-order matrices are defined as follows. Given a position operator $P(i, j)$, let A be a $n \times n$ matrix whose element A[i][j] is the number of times that points with gray level (intensity) g[i] occur, in the position specified by P, relative to points with gray level g[j]. Let C be the $n \times n$ matrix that is produced by dividing A with the total number of point pairs that satisfy P. C[i][j] is a measure of the joint probability that a pair of points satisfying P will have values g[i], g[j]. C is called a cooccurrence matrix defined by P. In particular, contrast features derived from the gray-scale cooccurrence matrices are defined as:

$$\sum_{i=0}^{M-1} \sum_{j=0, i \neq j}^{M-1} |i-j| C_{ij} \qquad (7)$$

These features represent a good measure for the contrast resolution of ultrasound images that can be used to weight the diffusion filters of ultrasound images.

B.3. Ultrasound Scatterer Density parameter

The Rayleigh distribution models the scatterer density where a large (infinite) number of uniformly distributed small size scatterers (compared to the wavelength of the ultrasound wave) are present. But this scenario is satisfied in very limited situations. In general, the "effective" number of scatterers is finite. Thus there is a need to model the situation with smaller scatterer density. A general distribution that accounts for small scatterer density was studied by V. Dutt in [5,19]. The envelope of the received backscattered signal A can be evaluated as:

$$p(A)=2\left(\frac{A}{2}\right)^{\alpha}\frac{b^{\alpha+1}}{\Gamma(\alpha)}K_{\alpha-1}(bA)\,, \tag{8}$$

where $b=\sqrt{\frac{4\alpha}{E\{A^2\}}}$ and $K_\beta()$ is the modified Bessel function of the second kind of order β.

This distribution is the so-called the K distribution. It gives a generalization of the Rayleigh distribution to account for small scatterer density. Dutt shows that the parameter α of the envelope of the amplitude density function could be treated as the "effective" number of scatterers per resolution cell. So, *we can propose to consider this parameter to be a measure of the speckle noisiness of the ultrasound images*. Next, we show how to evaluate this parameter from the K distribution moments.

The moments of K distributed data have a closed form expression as:

$$E\{A^{\eta}\}=\frac{(2\sigma^2)^{\eta/2}\Gamma(1+\eta/2)\Gamma(\alpha+\eta/2)}{\alpha^{\eta/2}\Gamma(\alpha)}\,. \tag{9}$$

Because the moments have closed form expressions, one can devise methods of estimating the parameters of K distributed data based on sample moments estimated from the data. Several methods have been proposed to estimate α from normalized moments [6]. A method that employs lower order moments is method of the second- and fourth-order moments. This method is as follows. Using (9), the normalized ratio of the fourth moment to the second moment squared can be written as:

$$\frac{E\{A^4\}}{E^2\{A^2\}}=2(1+\frac{1}{\alpha})\,. \tag{10}$$

This equation suggests an estimate for α using the sample fourth-order moment, μ_4, and second-order moment, μ_2 , as:

$$\hat{\alpha}=\frac{2}{\frac{\mu_4}{\mu_2}-2}\,, \tag{11}$$

where the sample moments are given by:

$$\mu_v=\frac{1}{N}\sum_{i=1}^{N}A_i^{v}\,, \tag{12}$$

where the A_i are the N samples of the envelope of the received backscattered signal used to estimate the parameters of the K distributed data.

III. RESULTS

We investigated the performance of nonlinear diffusion filters on reducing the speckle noise in ultrasound images (Liver image part acquired at 3.5Mhz showing hepatic vessels) using all possible combinations of three factors: the image gradient, the texture-based contrast, and the scatterer density. That is, we tested the performance using each factor alone, two at a time, and the three together. Suitable performance measures for evaluating the proposed nonlinear diffusion algorithm are the time evolution of the scatterer density in the whole image (a measure of noise removal property), the time evolution of the Canny edge detector map (a measure of edge detection and preservation property), and the visual interpretation.

A. Gradient weighted diffusion

Figs.1-b and 3 show the results of using the gradient only to weight diffusion of the noisy image in 1-a. We note that the noisy image has experienced progressive decrease in the scatterer density as time evolves (From 0.4445 to 0.2567). In addition, applying a standard edge detector, namely the Canny detector, the edge map proceeds from a truly vague map to a meaningful one. This result represents the basic Perona-Malik scheme against which the proposed filters are tested.

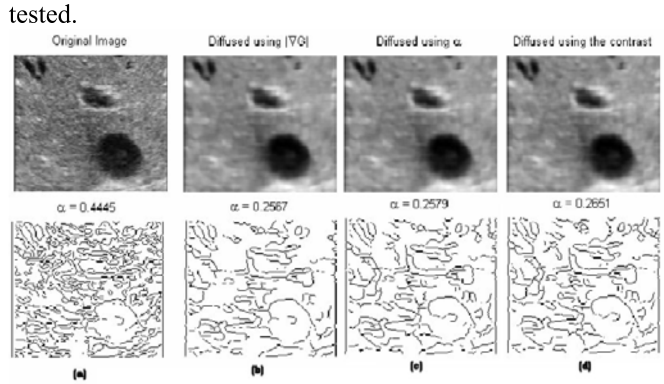

Fig.1. Original image (a), diffused using gradient only (b), using scatterer density only (c), using contrast only (d).

B. Scatterer Density weighted diffusion

Figs.1-c and 3 show the results of using the scatterer density only to weight diffusion of the noisy image in 1-a. We realize that the noisy image has experienced progressive decrease in the scatterer density as time evolves (From 0.4445 to 0.2579) but the final overall scatterer density (0.2579) is higher than that obtained using the gradient only. Also, the resulting edge map is slightly enhanced. This result shows us that the diffusion using the scatterer density can essentially perform as well as the diffusion depending on the gradient only. However, the gradient is an indispensable factor that should be present in the diffusivity with any new parameters for diffusion weighting.

C. Gradient and Scatterer Density weighted diffusion

In Figs.2-b and 3, we show the results of combining the gradient magnitude and scatterer density as multiplied factors into the diffusivity function. Then, we note that the noisy image has experienced more decrease in the scatterer density as time evolves (From 0.4445 to 0.2514) even more than that obtained using the gradient only. As well, the resulting edge map is enhanced remarkably. The result verifies our suggestion that the incorporation of the scatterer density in the diffusivity produces better diffusion results since the scatterer density can be regarded as a speckle noise measure.

We conclude that by incorporating the gradient magnitude multiplied by the scatterer density into the diffusivity function, we gain better performance – in terms of noise removal and edge preservation – than that obtained by any of the two factors alone. Fig.3. summarizes the time evolution of the overall scatterer density of the ultrasound image through diffusion using the gradient only, the scatterer density only, and the gradient multiplied by the scatterer density. Clearly, the gradient-times-scatterer-density combination displays the most reduction in scatterer density.

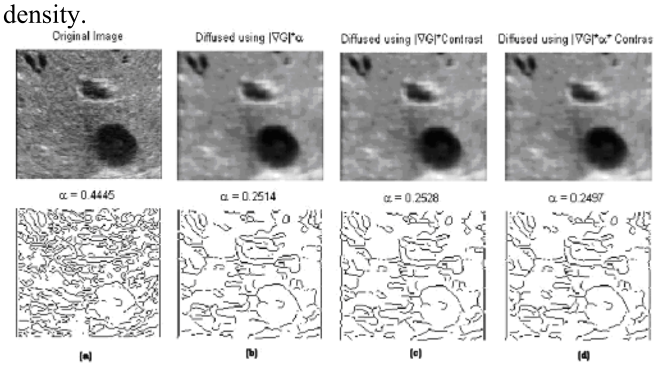

Fig.2. original image (a), diffused using gradient x scatterer density (b), using gradient x contrast (c), using gradient x scatterer density x contrast (d).

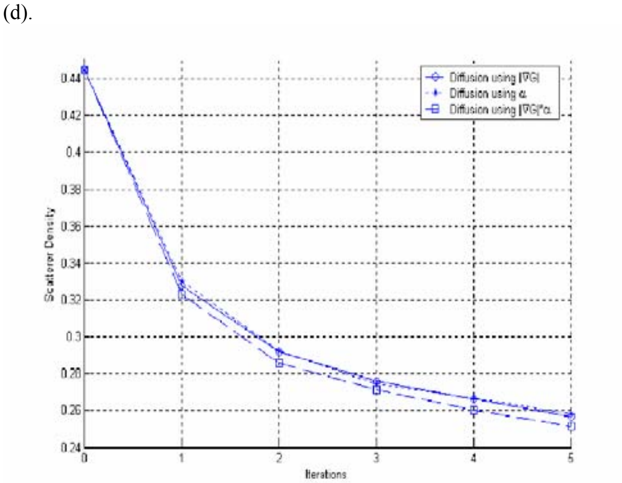

Fig.3. Time evolution of the overall scatterer density of the ultrasound image through diffusion using the gradient only, the scatterer density only, and the gradient multiplied by the scatterer density.

D. Contrast weighted diffusion

Figs.1-d and 4 show the results of using the scatterer density only to weight diffusion of the noisy image in 1-a. The noisy image has experienced progressive decrease in the scatterer density as time evolves (From 0.4445 to 0.2651) but the final overall scatterer density (0.2651) is higher than that obtained using either the gradient or the scatterer density. Also, the resulting edge map is slightly enhanced. We should not incorporate the contrast alone in the diffusivity function.

E. Gradient and Contrast weighted diffusion

In Figs.2-c and 4, we show the results of using a multiplicative combination of the gradient magnitude and the contrast to weight the diffusion filter of the noisy image in 2-a. Then, we note that the noisy image has experienced more decrease in the scatterer density as time evolves (From 0.4445 to 0.2528) even more than that obtained using the gradient only. As well, the resulting edge map is enhanced slightly. Fig.4. summarizes the time evolution of the overall scatterer density of the ultrasound image through diffusion using the gradient only, the contrast only, and the gradient multiplied by the contrast. Clearly, the gradient-times-contrast combination displays the most reduction in scatterer density.

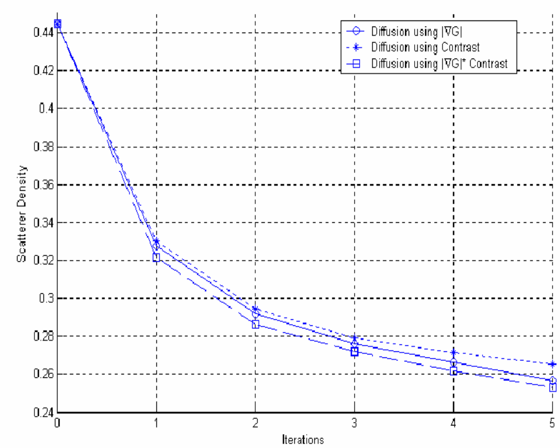

Fig.4. Time evolution of the overall scatterer density of the ultrasound image through diffusion using the gradient only, the contrast only, and the gradient multiplied by the contrast.

F. Gradient, Scatterer Density, and Contrast weighted diffusion

In Figs.2-d and 5, we examine the effect of multiplying the gradient magnitude, the scatterer density and the contrast. We note that the noisy image has experienced more decrease in the scatterer density as time evolves (From 0.4445 to 0.2497) even more than any other combination of factors. As well, the resulting edge map is enhanced slightly more than the previous results. We conclude that incorporating the gradient magnitude multiplied by the contrast and the scatterer density into the diffusivity function, we gain the best performance – in terms of noise removal and edge preservation – than that obtained by any other combination of factors. Fig.5. summarizes time evolution of the overall scatterer density of the image through diffusion using the gradient only, the gradient multiplied by the scatterer density, the gradient multiplied by the contrast, and the gradient multiplied by both of the scatterer density and the contrast. The all-factor combination has the lowest overall scatterer density and best edge map.

G. Scatter density as a measure for diffusion image quality

We calculated the traditional quality measures (RMSE, SNR, and PSNR) for the results of sections A-G. Table I shows the results of the RMSE, SNR, and PSNR in addition to the scatterer density we propose it to replace these traditional measures in ultrasound denoising process. However, the higher traditional measures do not always mean better quality. Scatterer density is better describing the quality of the images as there is a good relationship between the edge sharpness map and the value of the scatterer density as shown from sections A-G, the lower scatterer density the sharpened the detected edge map.

TABLE I
COMPARISON BETWEEN TRADITIONAL QUALITY MEASURES AND THE PROPOSED SCTTERER DENSITY MEASURE FOR ULTRASOUND DENOISING

Weighting Factors	RMSE	SNR(dB)	PSNR(dB)	A
Gradient	0.0471	4.368	26.538	0.2567
Scatter Density	0.0475	4.276	26.462	0.2579
Contrast	0.0464	4.527	26.658	0.2651
Gradient, Scatter Density	0.0484	4.039	26.303	0.2514
Gradient, Contrast	0.0477	4.192	26.424	0.2528
Gradient, Scatter Density, Contrast	0.0488	3.921	26.229	0.2497

The actual value of the traditional quality measures is not meaningful, but the comparison between two values for different diffused or reconstructed images gives one measure of quality. Using PSNR measure in image reconstruction, the MPEG committee used an informal threshold of 0.5 dB PSNR to decide whether to incorporate a coding optimization since they believed that any improvement of that magnitude would be visible. However, since the differences in PSNR values do not exceed 0.5 dB, we have an enhanced denoised visible image of good edge map sharpness that can be suitable as a preprocessing step for further segmentation or active contour processes of organs, tumors, or vessels.

H. Neural-Network implementation for proposed diffusion

We started to implement our novel diffusion filter using a standard multilayer backpropagation neural network. In this parallel-processing implementation, we trained the network using information extracted from the resulting diffusion filter. Taking the input to the network as a given noisy image shown in Fig.6.a and its targeted output as the one shown in Fig.6.b, we have got the following results as shown in Fig.7. We have extracted the grayscale, contrast, gradient, and scatterer density features for a 5*5 mask size and we used a combination of these 4 features as an input to the network of one hidden layer, 10 hidden-layer nodes, tanh sigmoidal activation function, 0.6 learning factor, 0.75 momentum term, and 10000 training epochs. Table II summarizes the NN output results in terms of the mean squared error (MSE) and the associated overall scatterer density measure. Fig.7. show this NN output for a test image of different grayscale information for another region in liver B-mode ultrasound. The shown results are very satisfactory for a 3D volumetric processing. These promising results suggest using this NN diffusion filter implementation for the parallel processing of a 3D image, which takes very short time in testing compared to the large time needed for 5 iterations of the diffusion filter.

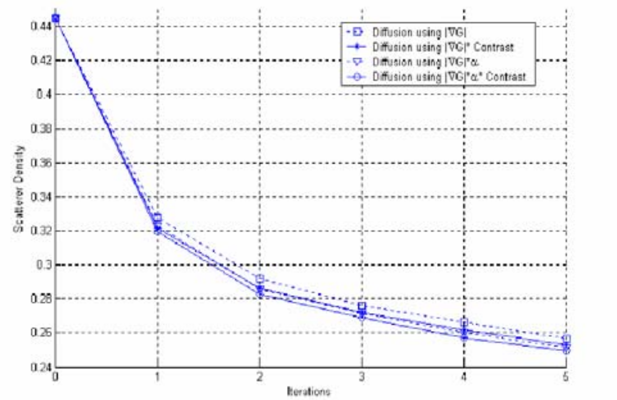

Fig.5. Time evolution of the overall scatterer density of the image through diffusion using the gradient only, the gradient multiplied by the scatterer density, the gradient multiplied by the contrast, and the gradient multiplied by both of the scatterer density and the contrast.

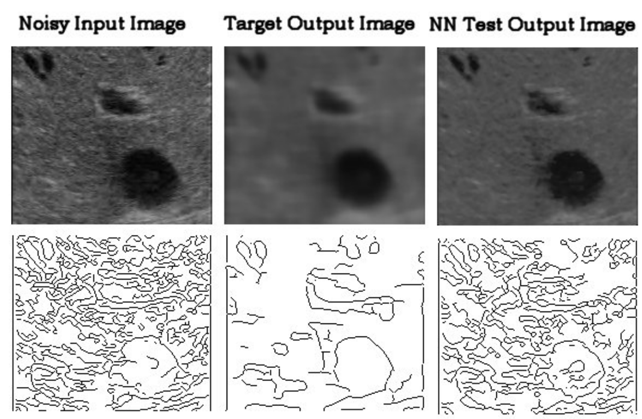

Fig.6. NN training using a noisy image as input, its diffused target image, and actual output image using the grayscale, contrast, gradient, and scatterer density features as input, and the output image is the grayscale value of the diffusion process.

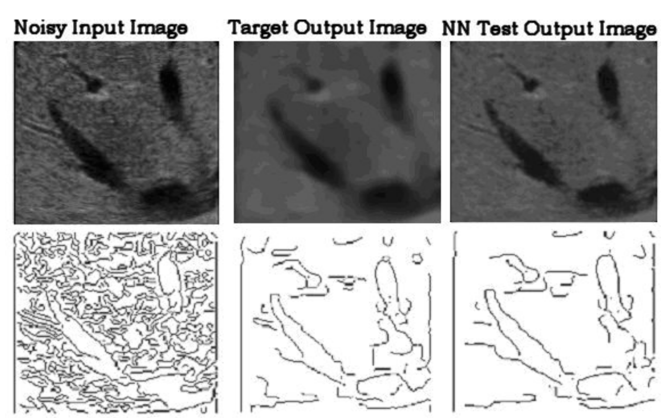

Fig.7. NN testing using a noisy image as input and tested result using the grayscale, contrast, gradient, and scatterer density features as input, the tested output image is the grayscale output from the NN test.

Using Gradient, Scatter Density, Contrast, and Gray-level as features for training with images of Fig. 6 and testing with images of Fig. 7, Table II and Table III show the comparison of the resulted quality measures between the input/target/test. Table III shows a better quality measures between the target and test image of Fig. 7 where we trained the network with the images of Fig. 6 and test the capability of the network to perform the diffusion process using images of Fig. 7.

TABLE II
TRAINING MSE, TESTING MSE, AND OVERALL SCATTERER DENSITIES FOR NETWORKS WITH DIFFERENT INPUT FEATURES

Network Input Features	Training MSE	Testing MSE	Overall Scatter Density
Gradient, Gray-level	8.7526	10.0863	0.1108
Scatter Density, Gray-level	9.0311	10.1907	0.0933
Gradient, Scatter Density, Gray-level	8.8063	10.0642	0.1019
Contrast, Gray-level	8.6903	9.9840	0.1020
Gradient, Contrast, Gray-level	8.9405	10.0975	0.1141
Gradient, Scatter Density, Contrast, Gray-level	8.4961	9.8367	0.1275

TABLE III
IMAGES RMSE, SNR, AND PSNR COMPARISON OF FIG. 7

Images	RMSE	SNR(dB)	PSNR(dB)
Between Input & Target	0.0478	3.164	26.411
Between Input & Test	0.0444	6.402	27.045
Between Target & Test	0.0322	6.567	29.818

IV. CONCLUSIONS

First, we proposed defining the scatterer density as a measure for denoising and image quality instead of the traditional MSE, RMSE, SNR, and PSNR where they do not have any visual meaning to judge a better ultrasound image quality. Second, we proposed using this scatterer density parameter in the nonlinear diffusion process alone or with gradient and/or contrast. We investigated the performance of nonlinear diffusion filters on reducing the speckle noise in ultrasound images using all possible combinations of these three factors: the image gradient, the texture-based contrast, and the scatterer density. From these experiments, we can confirm that the introduction of the contrast and the scatterer density into the diffusion process increases the good performance of the nonlinear diffusion algorithm in removing the speckle noise and preserving the important structures and edges of the image. The two new parameters introduced together to weight the nonlinear diffusion process make sense because they have a strong physical meaning, especially in ultrasound images. This method can be used in denoising other modality imaging such as MRI and CT. The proposed method showed a better quality of diffusion and better edge map where this method can be used further as a preprocessing phase before applying any segmentation or active contour model processes. Other factors of relative importance in modeling the speckle noise density and measuring the image quality could be tested similarly for potential use as weighting parameters of the ultrasound nonlinear diffusion filters. However, the overall computational time (~10 seconds for 128*128 image for 5 iterations) is long for volumetric processing in 3D. The bulk of this time is consumed in calculating the cooccurrence matrix, the gradient, and the scatterer density parameters. In calculating these parameters, we used traditional methods for calculation; despite fast accelerated computational methods exist in the literature, since our main task was to investigate the introduction of scatterer density in the evaluation of denoising process and weighting the nonlinear diffusion process. In the neural-network implementation, we used a noisy image as the input, its diffused image as the output then we tested the network with the different remaining noisy slices that were not used in network training and even from different images with different structures and different echographic patterns. Thus, we can accelerate the nonlinear diffusion process by implementing it as a neural filter based on multilayer backpropagation neural networks which are time-efficient in processing different volumetric slices.

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